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The description of the static electric and magnetic polarizabilities in the covariant Lagrangian formalism

N.V. Maksimenko* and S.A.Lukashevich†
Gomel State University
Sovetskay street, 102,
Gomel, 246699, Belarus

Within the covariant Lagrangian formalism the metric energy-momentum tensor and the equations of motion for spinor particles with polarizabilities in the electromagnetic field have been obtained. The contribution of static polarizabilities to the Compton scattering amplitude was determined.

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1. Introduction

At present time the description of the Compton scattering on hadrons is oriented on nonrelativistic Hamiltonian function [1, 2]. However for the extraction of the more essential experimental and theoretical information about hadron polarizabilities not only from the Compton scattering process but from other virtual and real two-photon process it is necessary to use the Lagrangian of electromagnetic field interaction with polarizable particle in covariant form.

The covariant Lagrangian of electromagnetic field interaction with polarizable particle was constructed in ref. [3–5]. So to find the covariant form of the Lagrangian in [3] the phenomenological formfactor approach for the construction of the Lagrangian covariant spin structures was used. The Lagrangian developed in ref. [4, 5] allowed us to determine the energy-momentum tensor of electromagnetic field interaction with polarizable particles as well as the the equations of motion and the Compton scattering amplitude.

2. Lagrangian

The total interaction Lagrangian of the spin-1/2 particles with the electromagnetic field will be consists from the Lagrangian for free electromagnetic field L_{e-m}, the spinor or Dirac's field L_D, the interaction Lagrangian of the free electromagnetic field with the Dirac's field $L_{int}$and the Lagrangian which considers electric and magnetic polarizabilities of particles $L_{\alpha_0\beta_0}$ [6]:

$$L_{total} = L_{e-m} + L_D + L_{int} + L_{\alpha_0\beta_0}. \tag{1}$$

So the total Lagrangian for the moving of the charged, polarizable, spinor particle in the electromagnetic field write out in the following form:

$$\mathcal{L} \;=\; -\frac{1}{4}F^2 + \frac{i}{2}\overline{\psi}\,\overset{\wedge}{\overset{\leftrightarrow}{\partial}}\,\psi - m\overline{\psi}\psi - e\overline{\psi}\widehat{A}\psi + L_{\alpha_0\beta_0}. \tag{2}$$

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*E-mail: maksimenko@gsu.unibel.by
†E-mail: lukashevich@gsu.unibel.by

The Lagrangian of electromagnetic field interaction with polarizable 1/2-spin particle can be presented as:

$$\mathcal{L}_{\alpha_0\beta_0} = \frac{2\pi}{m}\left[\alpha_0 F_{\mu\rho}F_{\sigma}{}^{\mu} - \beta_0 \widetilde{F}_{\mu\rho}\widetilde{F}_{\sigma}{}^{\mu}\right]\widetilde{\Theta}^{\rho\sigma}, \tag{3}$$

where $F^{\sigma\kappa}$ and $\widetilde{F}_{\mu\rho} = \frac{1}{2}\varepsilon_{\mu\rho\sigma\kappa}F^{\sigma\kappa}$ are usual and dual tensors of electromagnetic field, $\varepsilon_{0123} = -1$.

$$\widetilde{\Theta}^{\rho\sigma} = \frac{1}{2}(\Theta^{\rho\sigma} + \Theta^{\sigma\rho}), \tag{4}$$

$\Theta^{\rho\sigma}$ is the energy-momentum tensor of spinor field and given by

$$\Theta^{\rho\sigma} = \frac{i}{2}\overline{\psi}\gamma^{\rho}\overleftrightarrow{\partial^{\sigma}}\psi, \tag{5}$$

here $\overleftrightarrow{\partial_{\mu}} = \overrightarrow{\partial_{\mu}} - \overleftarrow{\partial_{\mu}}$, γ^{ρ} are the Dirac's matrixes.

The polarizabilities α and β introducing in the expression (3) are used in hadronic physic as static electric and magnetic polarizabilities.

To inserting the tensor

$$G^{(S)I\mu\nu} = -\frac{4\pi}{m}\left\{(\alpha_0 - \beta_0)\left[F^{\mu\sigma}\widetilde{\Theta}_{\sigma}{}^{\nu} - F^{\nu\sigma}\widetilde{\Theta}_{\sigma}{}^{\mu}\right] + \beta_0\widetilde{\Theta}_{\rho}{}^{\rho}F^{\mu\nu}\right\}, \tag{6}$$

the Lagrangian $\mathcal{L}_{\alpha_0\beta_0}$ can be rewritten as

$$\mathcal{L}_{\alpha_0\beta_0} = -\frac{1}{4}F_{\mu\nu}G^{(S)I\mu\nu}. \tag{7}$$

3. The equations of motion

For interaction of the spinor and electromagnetic fields the next system of Lagrange's equations are used [6]:

$$\left.\begin{aligned} -\frac{\partial L}{\partial A_{\mu}} + \partial_{\gamma}\frac{\partial L}{\partial(\partial_{\gamma}A_{\mu})} = 0, \\ -\frac{\partial L}{\partial\overline{\psi}} + \partial_{\gamma}\frac{\partial L}{\partial(\partial_{\gamma}\overline{\psi})} = 0, \\ -\frac{\partial L}{\partial\psi} + \partial_{\gamma}\frac{\partial L}{\partial(\partial_{\gamma}\psi)} = 0, \end{aligned}\right\} \tag{8}$$

where A_{μ} – is the vector-potential of the electromagnetic field, ψ and $\overline{\psi}$– are wave functions of spin-1/2 particles.

Taking into account the Lagrangian (2) and (8) we have got the equations of motion for the polarizable particles in the electromagnetic field

$$-i\gamma^{\alpha}(\partial_{\alpha}\psi) + \psi m + e\gamma_{\alpha}\psi A^{\alpha} + \frac{i\pi}{m}(2\gamma^{\alpha}(\partial_{\nu}\psi)\cdot K_{\alpha}^{\nu} + \gamma^{\alpha}\psi(\partial_{\nu}K_{\alpha}^{\nu})) = 0, \tag{9}$$

$$i(\partial_{\alpha}\overline{\psi})\gamma^{\alpha} - \overline{\psi}m + e\overline{\psi}\gamma_{\alpha}A^{\alpha} - \frac{i\pi}{m}(2(\partial_{\nu}\psi)\gamma^{\alpha}\cdot K_{\alpha}^{\nu} - \overline{\psi}\gamma^{\alpha}(\partial_{\nu}K_{\alpha}^{\nu})) = 0, \tag{10}$$

$$\begin{aligned} &-\partial_{\gamma}F^{\gamma\mu} - e(\overline{\psi}\gamma^{\mu}\psi) + \\ &+\partial_{\gamma}\left\{\frac{i\pi}{m}\left[(\alpha_0 - \beta_0)\left(F^{\mu\nu}(\overline{\psi}\gamma^{\gamma}\overleftrightarrow{\partial_{\nu}}\psi) - F^{\gamma\nu}(\overline{\psi}\gamma^{\mu}\overleftrightarrow{\partial_{\nu}}\psi) + \right.\right.\right. \\ &\left.\left.\left. + F^{\alpha\gamma}(\overline{\psi}\gamma_{\alpha}\overleftrightarrow{\partial^{\mu}}\psi) - F^{\alpha\mu}(\overline{\psi}\gamma_{\alpha}\overleftrightarrow{\partial^{\gamma}}\psi)\right) - 2\beta_0F^{\gamma\mu}(\overline{\psi}\gamma^{\nu}\overleftrightarrow{\partial_{\nu}}\psi)\right]\right\} = 0, \end{aligned} \tag{11}$$

where $K_{\mu}^{\nu}=\alpha_0 F_{\mu\rho}F^{\rho\nu}-\beta_0\tilde{F}_{\mu\rho}\tilde{F}^{\rho\nu}$.

The expression (11) we can be rewritten in a view

$$\partial_{\mu}F^{\mu\nu}=e\overline{\psi}\gamma^{\nu}\psi+j^{(M)\nu}, \tag{12}$$

where $j^{(M)\nu}=-\partial_{\mu}G^{(S)I\mu\nu}$.

4. Energy-momentum tensor for interaction of the electromagnetic field with spinor particle

To understand that the Lagrangian (2) is correct it is enough to define Hamiltonian for interaction of electromagnetic field with polarizable particle spin-1/2 and Compton scattering amplitude of this particle.

In the case of the spinor and electromagnetic fields the expression for energy-momentum tensor has the form [6]

$$T_{\nu}^{\mu}=\frac{\partial L}{\partial(\partial_{\mu}\psi)}(\partial_{\nu}\psi)+(\partial_{\nu}\overline{\psi})\frac{\partial L}{\partial(\partial_{\mu}\overline{\psi})}+(\partial_{\nu}A_{\rho})\frac{\partial L}{\partial(\partial_{\mu}A_{\rho})}-L\delta_{\nu}^{\mu}. \tag{13}$$

Using the Lagrangian (2) and antisymmetric tensor (6) the metric momentum-energy tensor we shall define as

$$\widetilde{T}^{\mu\nu} = \widetilde{\Theta}^{\mu\nu}+F_{\rho}{}^{\nu}F^{\mu\rho}+\frac{1}{4}g^{\mu\nu}F^2-\frac{e}{2}\overline{\psi}(\gamma^{\mu}A^{\nu}+\gamma^{\nu}A^{\mu})\psi+\widetilde{T}_I^{\mu\nu}, \tag{14}$$

where

$$\widetilde{T}_I^{\mu\nu}=F_{\rho}{}^{\nu}G_I^{(S)\mu\rho}+\frac{1}{4}g^{\mu\nu}(F_{\rho\sigma}G^{(S)I\rho\sigma}). \tag{15}$$

One can see from the expression (15) that for the particle in a rest its interaction energy appearing from polarizability will be have the form [2]

$$H_I=-2\pi(\alpha_0\mathbf{E}^2+\beta_0\mathbf{H}^2). \tag{16}$$

Taking into account (7) the scattering amplitude within second order over photon energy will give the contribution for electric and magnetic polarizabilities as

$$T_{fi}^{pol} = \frac{8\pi m\omega\omega'}{N(t)}\left[\mathbf{e}'^{*}\cdot\mathbf{e}\alpha_0+\mathbf{s}'^{*}\cdot\mathbf{s}\beta_0\right], \tag{17}$$

where $\mathbf{s}=\mathbf{n}\times\mathbf{e}$; $\mathbf{s}'^{*}=\mathbf{n}'\times\mathbf{e}'^{*}$, $\mathbf{e}$ and $\mathbf{e}'$ are the polarization vectors, ω_1 and ω_2 are the energies of the incident and scattered photons, $\mathbf{n}=\mathbf{k}/|\mathbf{k}|$, $\mathbf{n}'=\mathbf{k}'/|\mathbf{k}'|$.

The electric and magnetic polarizabilities have attracted a great number of researches to calculate these quantities from nucleon models. The majority of these calculations apply the concept of internal coordinates of the nucleon which are well defined in a nonrelativistic approach. In this non relativistic approach second order perturbation theory leads to the expressions [7]

$$\alpha' = 2\sum_{n\neq 0}\frac{|<n^{(i)}|D_z|0>|^2}{E_n^{(i)}-E_0^{(i)}}+Z^2\frac{e^2<r_E^2>}{3M}, \tag{18}$$

$$\beta' = 2\sum_{n\neq 0}\frac{|<n^{(i)}|M_z|0>|^2}{E_n^{(i)}-E_0^{(i)}}-e^2\sum_i\frac{q_i^2}{6m_i}<0|\rho_i^2|0>-\frac{<0|\mathbf{D}^2|0>}{2M}. \tag{19}$$

These equations contain the retardation correction $\triangle\alpha = Z^2e^2 < r_E^2 > /3M$ of the electric polarizability and the diamagnetic susceptibility $\beta_{dia} = -e^2\sum_i(q_i^2/6m_i) < 0|\rho_i^2|0 > - < 0|D^2|0 > /2M$ in addition to the leading terms coming from second-order perturbation theory in the long wave -length limit. In (20) and (21) Z and M are the charge number and total mass, respectively, of the hadron and r_E^2 the square of the quadratic charge radius. The quantity $\mathbf{D}$ is the electric dipole moment and D_z and M_z the z-components of the electric and magnetic dipole moments, respectively. The quantities q_i, m_i and ρ_i are the charge fraction, the mass and the internal coordinate of the constituents inside the hadron. Resently, it has been shown that these relations contain large uncertainties, especially in the r_E^2 dependent retardation correction because there are other relativistic terms of at least the same order.

We would like to say that our electric α_0 and magnetic β_0 polarizabilities doesn't depend on the square of the quadratic charge radius r_E^2, i.e.

$$\alpha_0 = 2\sum_{n\neq 0}\frac{| < n^{(i)}|D_z|0 > |^2}{E_n^{(i)} - E_0^{(i)}}, \tag{20}$$

$$\beta_0 = 2\sum_{n\neq 0}\frac{| < n^{(i)}|M_z|0 > |^2}{E_n^{(i)} - E_0^{(i)}}, \tag{21}$$

so they are the static one.

5. Conclusion

The covariant Lagrangian constructed on the basis of the relativistic electrodynamics of continuous media formalism and main relativistic quantum field theory principles have let us to determine metric energy-momentum tensor as well as the equations of motion for spinor particles with polarizabilities in the electromagnetic field. The contribution of electric and magnetic polarizabilities to the Compton scattering amplitude have shown that they are the static one.

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