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Instanton tunneling suppression in kicked double well potential

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Kicked one-dimensional quantum system with double-well potential is considered. Monte Carlo simulations of instanton transitions between potential wells demonstrate exponential instanton tunneling suppression when either perturbation strength or frequency increase.

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1. Introduction

Semiclassical properties of systems with mixed classical dynamics is a reach rapidly developing field of research. One of interesting results obtained in this direction is a chaos assisted tunneling. It was shown that the structure of the classical phase space of Hamiltonian systems can influence such purely quantum processes as the tunneling [1, 2]. It was demonstrated in numerical simulations that existence of chaotic motion region in the classical phase space of the system can increase or decrease tunneling rate by several orders of magnitude [1, 3]. Typically one considers tunneling between KAM-tori embedded into the "chaotic sea". The region of chaotic motion affects tunneling rate because compared to direct tunneling between tori it is easier for the system to penetrate primarily into the chaotic region, to travel then along some classically allowed path and to tunnel finally to another KAM-torus [4, 5].

Chaos assisted tunneling phenomenon as well as the closely related coherent destruction of tunneling were experimentally observed in a number of real physical systems. The observation of the chaos assisted tunneling between whispering gallery-type modes of microwave cavity having the form of the annular billiard was reported in the Ref. [6]. The study of the dynamical tunneling in the samples of cold cesium atoms placed in an amplitude-modulated standing wave of light provided evidences for chaos-assisted (three-state) tunneling as well [7]. Recently, the coherent destruction of tunneling was visualized in the system of two coupled periodically curved optical waveguides [8].

The most popular methods which are used to investigate the chaos assisted tunneling are numerical methods based on Floquet theory [9]. Among other approaches to chaos-assisted tunneling we would like to mention the path integral approach for billiard systems [10] and quantum mechanical amplitudes in complex configuration space [11]. In this paper we will consider the original approach based on instanton technique, which was proposed in [12–14] and numerically tested in [15, 16].

Instanton is a universal term to describe quantum transition between two topologically distinct vacuum states of quantum system. In classical theory the system can not penetrate potential or dynamical barier, but in quantum theory such transitions may occur due to tunneling effect. It is known that tunneling processes can be described semiclassically using path

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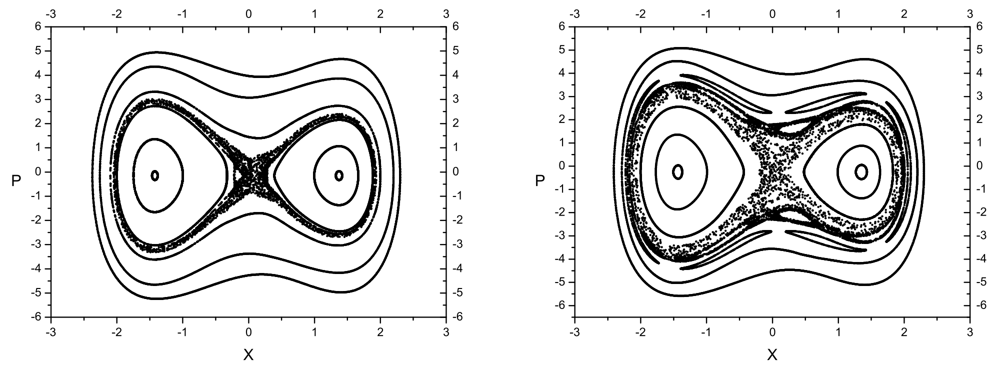

(a) (b)

FIG. 1. (a) Phase portrait of the system (1) for $\epsilon = 0.3$; (b) Phase portrait of the system (1) for $\epsilon = 0.5$. Well developed stochastic layer is seen. The perturbations are strong enough to break the non-perturbed Hamiltonian symmetry $x \to -x$. Parameter $f = 1.4$ is the same in both cases.

integrals in Euclidean (imaginary) time. In this case instantons are soliton-like solutions of the Euclidean equations of motion with a finite action. For example, in Euclidean Yang-Mills theory distinct vacuum states are the states of different Chern-Symons classes, and instanton solutions emerge due to topologically nontrivial boundary conditions at infinity. In this paper we will use a much simpler system to investigate the connection between the tunneling, instantons and chaos, namely the kicked system with double well potential.

2. Formulation of the problem and Monte Carlo method used

We consider the kicked double well system with the following Hamiltonian:

$$H = \frac{\dot{x}^2}{2} + \left(x^2 - f^2\right)^2 + \epsilon x \sum_{n=-\infty}^{+\infty} \delta(t - nT). \tag{1}$$

Here f is the distance between potential wells, ϵ is the perturbation strength, and T is the period of kicks. Phase portraits of the system for two different values of ϵ are shown in Fig. 1. Quantum mechanical system (1) without kicks ($\epsilon = 0$) was considered in a huge number of publications. In this work the paper [17] has to be mentioned, where time independent case $\epsilon = 0$ was entirely studied by mean of the Monte Carlo simulations and instanton technique. Similar methods and approaches are used in this paper in order to address the time dependent system (1). Numerical simulations of quantum tunneling dynamics of the system (1) are based on Monte Carlo method, namely, Metropolis algorithm is used. All calculations are performed in imaginary (Euclidean) time.

Numerical evaluation of the tunneling amplitude and average values of quantum mechanical observables are achieved via summation over all possible configurations $x(\tau)$ weighted with the exponent of their Euclidean action. The last is evaluated on imaginary time grid with the spacing between neighbor sites a as

$$S = \sum_{i=1}^{n} \left[\frac{1}{2a} \left(x_i - x_{i-1}\right)^2 + a \left(x_i^2 - f^2\right)^2 + \epsilon x_{i=[\tau/T]} \right], \tag{2}$$

where the brackets $[\ldots]$ denote the integer part of real number, $x_i = x(\tau_i)$ and discrete Euclidean time grid is given by the expression $\tau_i = ia$, $i = 1, \ldots, n$. Periodic boundary condition $x_0 = x_n$

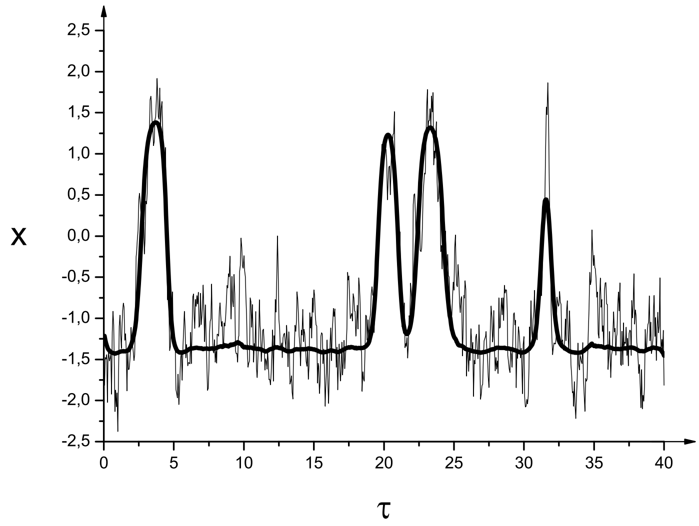

FIG. 2. The example of the typical configuration obtained by means of Metropolis algorithm (thin curve) vs the same configuration after the cooling procedure (thick curve). The Euclidean time grid step equals 0.05.

is used in numerical simulations. The number of time grid sites is equal to 800 in all numerical simulations in this paper.

In the framework of Monte Carlo algorithm used the successive configuration $\{x_i\}^{(k+1)}$ is generated by means of the Metropolis update of the current configuration $\{x_i\}^{(k)}$. It is achieved via a trial sweep performed for every lattice site $x_i^{(k+1)} = x_i^{(k)} + \delta x_i$. Here δx_i is a random number independently generated for the site labeled i. This trial update is accepted with the probability:

$$P\left(\{x_i\}^{(k)} \rightarrow \{x_i\}^{(k+1)}\right) = \min\{\exp(\Delta S), 1\}, \tag{3}$$

where ΔS is the difference between Euclidean actions of the new (trial) and current configurations. This mechanism ensures that we accept the updates reducing the action with the unit probability whereas the updates raising the action are partially rejected (usually with the probability around 0.5). It guarantees that in long-term outlook we are approaching to the configurations with the minimal Euclidean action yielding the dominating contribution to the tunneling amplitude. The sweeps raising the action are not rejected completely, because they provide "Boltzmann" distribution $\exp(-S)$ in the path integral, i.e. they are responsible for the taking into account of quantum fluctuations. The example of the typical configuration obtained by these means is shown in Fig. 2. From this figure it is seen (look at the thin curve) that a typical path is a superposition of two components, the first one represents rapid short time scale oscillations with the frequency $\sim \omega = 2\sqrt{2}f$ related to quantum fluctuations. The second one is the series of transitions between the potential minima and it is related to tunneling events, i.e. instantons. In order to study these tunneling events in more details the short scale fluctuations have to be removed. It can be achieved applying well known "cooling" technique [17, 18]. In the framework of this method Metropolis updates that lower Euclidean action are accepted only. It drives configuration to the nearest classical solution and, thus, allows to study the instanton content of the configuration. The thick curve in Fig. 2 is the same configuration as plotted by the thin curve but after the 10 cooling sweeps application. In order to extract the instanton content the simple "sum ansatz" is applied to the cooled configuration:

$$x(\tau) = \sum_i Q_i x_{inst}(\tau - \tau_i), \tag{4}$$

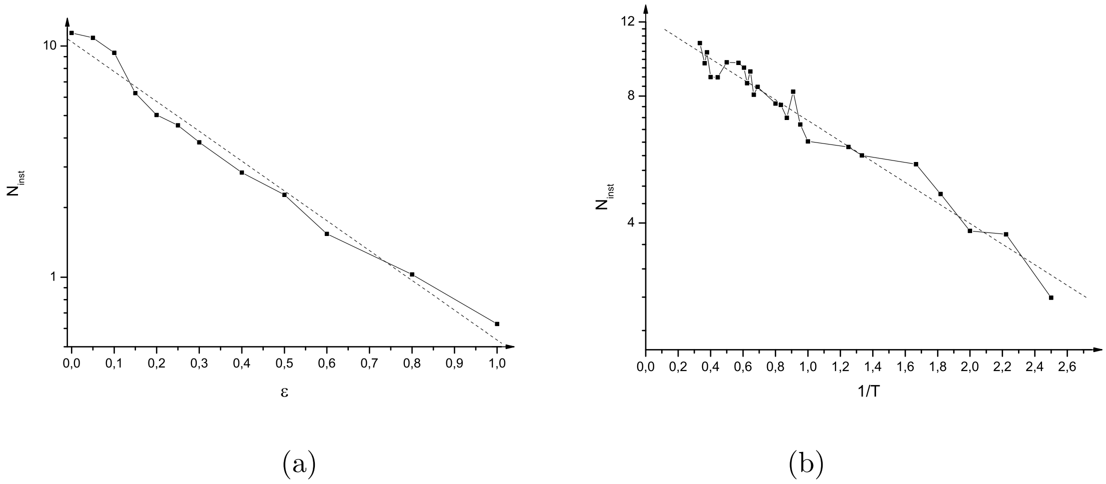

FIG. 3. (a) Logarithmic plot of the number of (anti-)instanton transitions vs. the perturbation strength, the perturbation frequency $\nu = 2$; (b) logarithmic plot of the number of (anti-)instanton transitions vs. the perturbation frequency $\nu = 1/T$, the perturbation strength $\epsilon = 0.3$. The number of the cooling sweeps equals 10 in both cases.

where $Q_i = \pm 1$ is the topological charge of the instanton/anti-instanton, $x_{inst}(\tau - \tau_i)$ is the instanton solution placed at τ_i and the index i labels the instantons and anti-instantons. Thus the instanton and anti-instanton locations can be extracted as the zero crossings of the cooled configuration. In the simulations bellow the number of the cooling sweeps is accepted to be equal to 10, because this value leads to a good agreement between numerical and analytical results for the non-perturbed system.

3. Results of numerical simulations

In this paper the dependence of the instanton/anti-instanton tunneling transitions number on the perturbation strength and frequency was numerically studied. The parameter f was accepted to be equal to 1.4 throughout this study and both the perturbation strength ϵ and frequency $\nu = 1/T$ have been varying. The results of the numerical simulations are shown in Fig. 3. Logarithmic plot of the number of (anti-) instanton transitions versus the perturbation strength is shown in the Fig. 3(a). It is seen that this dependence can be well approximated by the straight line, standard deviation equals 0.06. Thus the number of (anti-)instanton transitions exponentially decreases when the perturbation strength rises with the perturbation frequency being fixed. Logarithmic plot of the number of (anti-)instanton transitions versus the perturbation frequency ν is shown in the Fig. 3(b). It is seen that this dependence is well approximated by the straight line as well with the standard deviation 0.03. The perturbation strength ϵ is fixed and equal to 0.3. Thus the number of (anti-)instanton transitions exponentially decreases when the perturbation frequency rises with the perturbation strength being fixed. The number of cooling sweeps is equal to 10 in both cases.

4. Conclusion

The dependence of the number of the instanton/anti-instanton tunneling transitions both on the perturbation strength and frequency was numerically studied. Monte Carlo technique was applied for this purpose. Namely, it was demonstrated that the number of (anti-)instanton transitions exponentially decreases when either the perturbation strength or the frequency increase with another parameter being fixed. These results in freezing of the wave packet as a whole in one of the potential wells. However small magnitude tunneling fluctuations between wells are not excluded.

5. Acknowledgements

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