Nonlinear Dynamics and Applications. Vol. 14 (2007) 85 - 90

Double logarithmical corrections to beam asymmetry in polarized electron-proton scattering

E. Kuraev, S. Bakmaev, V. V. Bytev, and Yu. M. Bystritskiy
JINR-BLTP, 141980 Dubna, Moscow region, Russian Federation

E. Tomasi-Gustafsson
DAPNIA/SPhN, CEA/Saclay, 91191 Gif-sur-Yvette Cedex, France

The up-down asymmetry in transversally polarized electron proton scattering is induced by the interference between one and two photon exchange amplitudes. Inelastic intermediate hadronic states (different from one-proton state) of the two photon exchange amplitude give rise to contributions containing the square of "large logarithm" (logarithm of the ratio of the transferred momentum to the electron mass). We investigate the presence of such contributions in higher orders of perturbation theory. The relation with the case of zero transfer momentum is explicitly given. The mechanism of cancellation of infrared singularities is discussed.

PACS numbers: 12.38.-t, 13.40.-f, 13.60.-r
Keywords: electron-proton scattering, polarization, infrared singularities

1. Introduction

The beam asymmetry due to transversal polarization of an electron beam scattered on (unpolarized) protons is a pure quantum effect arising from the interference of the Born amplitude (with one photon exchange) and the imaginary part of the two-photon exchange amplitude (Fig. 1).

The corresponding contribution to the differential cross section as well as to the beam asymmetry is proportional to the electron mass. Therefore, the presence of this contribution does not contradict the Kinoshita-Lee-Nauenberg theorem [1], about cancellation of mass singularities, since the corresponding cross sections are suppressed by the lepton mass.

We show that the main contribution arises from the kinematical region of loop momenta when the energy of the electron in the intermediate state, being on mass shell, is small in the reference frame of the center of mass of the initial particles.

The kinematics is determined by the conservation laws

$$e^-(a, p_1) + P(p) + (\gamma) \to e^-(p_1'') + X + (\gamma) \to e(p_1') + P(p') + (\gamma) \tag{1}$$

where a is electron spin. In general, the emission of real photons must be considered, in order to avoid infrared divergences, when higher orders of perturbation theory (PT) are taken into

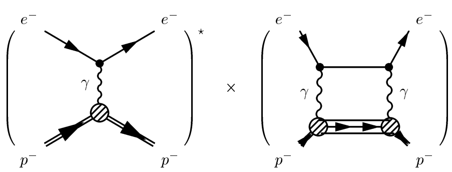

FIG. 1: The beam asymmetry due to transversal polarization of an electron beam in lowest order

account. For the case of one proton intermediate state in the two-photon exchange amplitude (TPE) the energies of the initial intermediate state and the final elastically scattered electrons are equal

$$p_{10} = p''_{10} = p'_{10} = \epsilon = \frac{s - M^2}{2\sqrt{s}}, \quad s = (p_1 + p)^2,$$
$$t = (p_1 - p'_1)^2 = -2\epsilon^2(1 - c), \quad p_1^2 = (p'_1)^2 = m^2,$$
$$p^2 = (p')^2 = M^2, \tag{2}$$

where $c = \cos\theta$ and θ is the scattering angle in the center of mass frame. The absolute values of the photon momenta squared, in TPE amplitude, can reach zero:

$$\left|t_1\right| = \left|(p_1 - p''_1)^2\right| = 2\epsilon^2(1 - c_1),$$
$$\left|t_2\right| = \left|(p'_1 - p''_1)^2\right| = 2\epsilon^2(1 - c_2), \tag{3}$$

where $c_{1,2}$ are the cosines of the angles between the initial and intermediate electron momenta and the intermediate and final ones, respectively.

For the case of inelastic hadronic intermediate state (for instance a nucleon and a pion) the energy of the electron in the intermediate state ϵ'' does not exceed $\epsilon : m < \epsilon'' < \epsilon$. The exchanged photon momenta squared become:

$$t_{1,2} = -2\epsilon\epsilon''(1 - bc_{1,2}), \quad 1 - b^2 = \frac{m^2}{(\epsilon'')^2}(1 - \frac{\epsilon''}{\epsilon})^2. \tag{4}$$

The main contribution arises from two regions $-t_1 \ll t_2 = t$ and $-t_2 \ll t_1 = t$.

Moreover, we will show that the energy of the intermediate electron is much lower than the electron energy corresponding to the elastic case $\epsilon'' \ll \epsilon$.

Neglecting the dependence on p''_1 from the remaining part of the amplitude we find the main (double-logarithmical, or DL) asymptotic behavior of the amplitude:

$$(M^*_{box} M_{born})^{(DL)} \approx \int \frac{\epsilon'' \mathrm{d}\epsilon'' \mathrm{d}O''}{2\pi t_1 t_2} \approx \frac{-1}{4t} L^2. \tag{5}$$

We do not distinguish here the two kinds of "large logarithms"

$$L_s = \ln\frac{s}{m^2} - i\pi, \quad L_t = \ln\frac{-t}{m^2} = \ln\frac{2\epsilon^2(1 - c)}{m^2}. \tag{6}$$

The result of exact calculation consists in the replacement

$$L^2 \to L_t L_s. \tag{7}$$

2. Calculation of Double Logarithm correction

Let us investigate the question about the size of radiative corrections to the cross section for the scattering of transversally polarized electron and to the relevant beam asymmetry. The corrections in lowest order can be of several types (see figs. 2, 3, 4, 6, 7).

Let us consider first the radiative corrections (RC) at the lowest order. The contribution from the emission of virtual photons is twofold. Firstly, it is due to the vertex functions in the kinematics where both electrons are on mass shell and the photon mass squared is negative

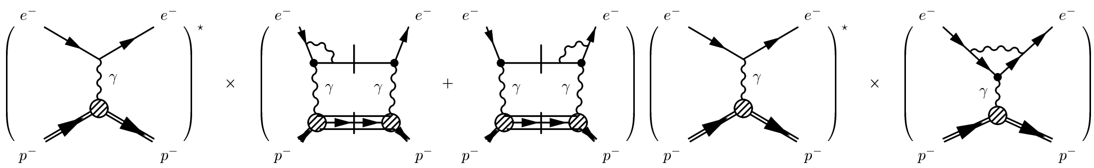

FIG. 2: Vertex corrections δ_v which contribute to the asymmetry

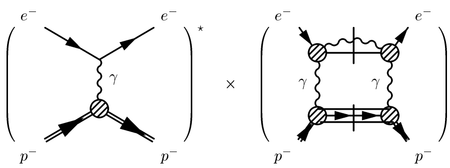

FIG. 3: Inner bremsstrahlung in two photon diagram

and large (in absolute value) compared to the electron mass squared (see Fig. 2). The main contribution arises from the Dirac form factor. In the scattering channel we have:

$$F_1(q^2) = 1 + \frac{\alpha}{\pi}\left[\ln\frac{m}{\lambda}(1-L_q) - 1 + \frac{3}{4}L_q - \frac{1}{4}L_q^2 + \frac{\pi^2}{12}\right]$$
$$= 1 + \frac{\alpha}{\pi}\delta_v(q^2),$$
$$q^2 = t, t_1, t_2, \quad L_q = L_t, L_1, L_2,$$
$$L_1 = \ln\frac{-t_1}{m^2}, \quad L_2 = \ln\frac{-t_2}{m^2}. \tag{8}$$

The second class of contributions arise from emission and absorption of real photons as an intermediate state of the leptonic block. Let firstly restrict our considerations to the emission of soft real intermediate photons (see Fig. 3).

For the corresponding contribution we have

$$\frac{\alpha}{\pi}\delta_s = -\frac{4\pi\alpha}{16\pi^3}\int\frac{\mathrm{d}^3k}{\omega}$$
$$\times\left(-\frac{p_1}{p_1k} + \frac{p_1''}{p_1''k}\right)\left(\frac{p_1''}{p_1''k} - \frac{p_1'}{p_1'k}\right)\Big|_{\omega<\Delta_1},$$
$$\Delta_1 \ll \epsilon. \tag{9}$$

Using the expressions

$$-\frac{4\pi\alpha}{16\pi^3}\int\frac{\mathrm{d}^3k}{\omega}\frac{m^2}{(p_1''k)^2}\Big|_{\omega<\Delta_1} = -\frac{\alpha}{\pi}\ln\frac{m\Delta_1}{\lambda\epsilon''},$$
$$\frac{4\pi\alpha}{16\pi^3}\int\frac{\mathrm{d}^3k}{\omega}\frac{2p_1p_1''}{(p_1k)(p_1''k)}\Big|_{\omega<\Delta_1} = \frac{\alpha}{2\pi}\left[L_1\ln(\frac{m^2\Delta_1^2}{\lambda^2\epsilon\epsilon''})\right.$$
$$\left.+\frac{1}{2}L_1^2 - \frac{1}{2}\ln^2\left(\frac{\epsilon''}{\epsilon}\right) - \frac{\pi^2}{3} + \mathrm{Li}_2\left(\frac{1+c_1}{2}\right)\right], \tag{10}$$

where λ is a fictitious "photon mass", the resulting contribution

$$\delta_{vs} = \delta_v(t) + \delta_v(t_1) + \delta_v(t_2) + \delta_s \tag{11}$$

suffers from infrared divergences. To remove these divergencies, we must take into account the inelastic process of electron-proton scattering with emission of additional soft (or hard) real

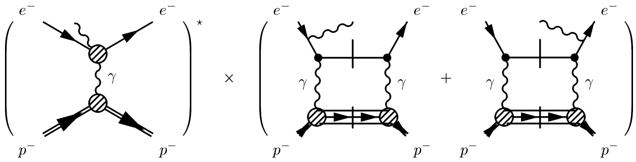

FIG. 4: Inelastic two photon contribution δ_s^{inel}

photons by initial and final electrons (see Fig 4):

$$\frac{\alpha}{\pi}\delta_s^{inel} = -\frac{4\pi\alpha}{16\pi^3}\int\frac{\mathrm{d}^3k}{\omega}\left(-\frac{p_1}{p_1k}+\frac{p_1'}{p_1'k}\right)^2\Big|_{\omega<\Delta_2}, \quad \Delta_2 \ll \epsilon. \tag{12}$$

The total sum $\delta = \delta_{vs} + \delta_s^{inel}$ is free from infrared divergences:

$$\delta = 2(L_t-1)\ln\frac{\Delta_2}{\epsilon}+\frac{1}{2}(L_1+L_2)\ln\frac{\Delta_1^2}{\epsilon\epsilon''}-\ln\frac{\Delta_1}{\epsilon''} -L_t\ln\frac{\Delta_1}{\epsilon}+\frac{3}{4}(L_t+L_1+L_2)-3-\frac{\pi^2}{4} +\frac{1}{2}\left[\mathrm{Li}_2\left(\frac{1+c_1}{2}\right)+\mathrm{Li}_2\left(\frac{1+c_2}{2}\right)+\mathrm{Li}_2\left(\frac{1+c}{2}\right)\right] -\frac{1}{2}\ln^2(\frac{\epsilon''}{\epsilon}). \tag{13}$$

As the dominant (DL) contribution arises from the region of intermediate state with a soft lepton in presence of photons, we can generalize the above result including the emission of "hard" internal and external photons. This can be realized by the choice

$$\Delta_1 \sim \Delta_2 \sim \epsilon''. \tag{14}$$

In this case the hadronic block remains the same as for the lowest order of PT.

In DL approximation we have

$$\delta^{DL} = \left[L_t+\frac{1}{2}(L_1+L_2)\right]\ln\frac{\epsilon''}{\epsilon}-\frac{1}{2}\ln^2\frac{\epsilon''}{\epsilon}. \tag{15}$$

Up to now we calculate the contribution of box diagram to the matrix element. For the calculation of the asymmetry we must to take into account imaginary pairs of the contribution. By general arguments of analiticity [7] the matrix diagram with two-photon exchange and insertion of vertex function with correction of α^n order in DL approximation is proportional to $L_sL_t^{2n-1}$.

It is naturally to expect (keeping in mind the arguments in favor of exponentiation of soft photon emission contributions proved by Yenne, Frautchi and Suura [2]) that this result can be generalized to all orders of PT:

$$R = \frac{\mathbf{Im}(M_{box}^*M_{born})^{(DL)+corr}}{\mathbf{Im}(M_{box}^*M_{born})^{(DL)}} = \left[\int_0^{L_t}\frac{(1-c)\,\mathrm{d}O''\,\mathrm{d}l}{2\pi(1-bc_1)(1-bc_2)}\exp\left(\frac{\alpha}{\pi}\delta^{DL}\right)\right] \times\left[\int_0^{L_t}\frac{(1-c)\,\mathrm{d}O''\,\mathrm{d}l}{2\pi(1-bc_1)(1-bc_2)}\right]^{-1}. \tag{16}$$

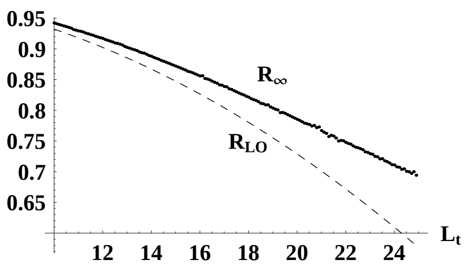

FIG. 5. Numerical results for R, Eq. (18). R_{LO} (dased line) is the result of the calculation with RC in lowest order of PT, R_∞ (dotted line) when all orders of PT are taken into account. We assume $c = \frac{1}{2}$.

We can use here

$$\mathrm{d}O'' = \frac{2\mathrm{d}c_1\mathrm{d}c_2}{\sqrt{1-c_1^2-c_2^2-c^2+2cc_1c_2}},$$
$$\delta^{DL} = \frac{3}{4}L_t l + \frac{1}{8}l^2 - \frac{7}{8}L_t^2$$
$$+\frac{1}{4}(l-L_t)\left[\ln(1-bc_1)+\ln(1-bc_2)\right],$$
$$l = \ln\frac{2(\epsilon'')^2(1-c)}{m^2}. \quad (17)$$

The calculation in the lowest order of PT leads to

$$R_{LO} \approx 1 - \frac{\alpha}{\pi}\frac{7L_t^2}{24}. \quad (18)$$

The numerical results for R in the lowest order, R_{LO}, and in higher orders, R_∞, are presented in Fig. 5. One can see they are sizable and should be taken into account.

The asymmetry at the lowest order of PT has been calculated in previous papers, (see for instance [3]).

Mass suppressed amplitudes connected with Higgs production and decay in DL approximation were calculated in the paper [8].

3. Conclusion

We have calculated higher order contributions to the asymmetry for elastic scattering of transversally polarized electrons on unpolarized protons. In particular we have shown that the double logarithmic corrections arising from the inelastic intermediate state in the two photon exchange amplitude can not be neglected.

The contribution to the imaginary part of the amplitude from the square of the box diagram (Fig. 6) is of order $(\alpha/\pi)^2 L$, and can be omitted in DL approximation. This holds also for the interference of the born diagram with the two loop box diagram (Fig. 7) [6].

We note that the limiting case $t \to 0$ can not be obtained using the approach described above. Contrary to the hadronic block, where the $t \to 0$ limit can be put smoothly, the leptonic block drastically depends on the parameter $(-t/m^2)$. For the $t = 0$ case, the exact cancellation of RC related to the internal emission of virtual and real photons takes place, as it was shown in Appendix D of [4].

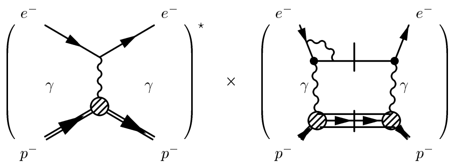

FIG. 6: Square box contribution to the asymmetry

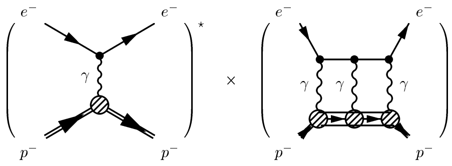

FIG. 7. Contribution to the asymmetry of the interference of the Born diagram with the two loop box diagram

The presence of infrared singularities in RC to the impact factor of the electron was previously noted in the paper of one of us [5]. In the present paper, the way to eliminate such singularities is explicitly shown.

Acknowledgments. We are grateful for valuable remarks to the A. Kobushkin and D. Borisyuk. Two of us (E.A.Kuraev, V.V. Bytev) are partially supported by the INTAS grant 05-1000008-8328, MK-2952.2006.2 and BRFBR (contract F06D-002).

The authors are grateful to Professor I.V. Puzynin for valuable discussions. This work has been partially supported by a grant of the Russian Foundation for Basic Research 03-01-00657.

References

[1] T. Kinoshita, J. Math. Phys. **3** (1962) 650.
T. D. Lee and M. Nauenberg, Phys. Rev. **133** (1964) B1549.

[2] D. R. Yennie, S. C. Frautschi and H. Suura, Annals Phys. **13** (1961) 379.

[3] A. V. Afanasev and N. P. Merenkov, Phys. Rev. D **70** (2004) 073002
D. Borisyuk and A. Kobushkin, Phys. Rev. C **73** (2006) 045210

[4] V. N. Baier, E. A. Kuraev, V. S. Fadin and V. A. Khoze, Phys. Rept. **78** (1981) 293.

[5] E. A. Kuraev, L. N. Lipatov and T. V. Shishkina, J. Exp. Theor. Phys. **92** (2001) 203 [Zh. Eksp. Teor. Fiz. **92** (2001) 236]

[6] A.I. Akhiezer and V. B. Berestezki, "Quantum Electrodynamics", Moscow Nauka (1981).

[7] R. J. Eden, P.V. Landshoff, D.I. Olive and J.C. Polkiinghorne, " The Analytic S-Matrix", Cambridge University (1966)

[8] V. S. Fadin, V. A. Khoze and A. D. Martin, Phys. Rev. D **56** (1997) 484
M. Kotsky and O. Yakovlev (unpublished).