Information-theoretic approach to the study of the radionuclide composition formation in case of nuclear accident at a nuclear power plant

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To develop a model of a nuclear accident we used information theory conceptions and represented this complex process in term of coding, transfer and decoding of information. We consider the unique structure of fission product decay chains as Markov process that generates a code for radionuclide composition formation.

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1. Introduction

The subject we would like to discuss in the presented investigation is concerned with the central problem of nuclear accident. This problem is referred to the preparedness of adequate emergency protection plan and of effective execution of this plan. By this the effective strategy of population protection and the related problem of the control system in this context are meant. Prior to discuss the essence of the problem two remarks need to be made. The first. National emergency control system that is now in force in the Republic of Belarus consists of the radiation-monitoring network in connection with man-machine computerized decision support system. The system provides radiation protection against the negative consequences in case of accident at a number of Nuclear Power Plant (NPP) sited near the State border of Belarus in Lithuania, Ukraine and Russia. The second. In accordance with the Radiation Standards of Belarus [1], radiation accident is regarded as loss of control of the NPP caused by an accidental event. A loss of control initiated by an uncontrollable delayed neutron kinetic may disturb energy balance of the NPP and lead to severe accident. The high-power burst arising from the disturbed energy balance is able to destroy safety barriers of the NPP. Once the radioactive release into the environment has occurred, an expected population dose prediction is the major problem of the emergency control system. Accidental events produce unexpected, unpredicted situation before the emergency decision-making team. The control system usually fails to operate efficiently when faced with uncertainty and default of realistic data [2].

As proved in general control theory, to enhance the functional quality required the control system must be adaptive. Firstly, it must be capable of estimating the unpredictable features of the controlled object during the control process, and, secondly, must be able to use the information provided by estimation for specific action decision, just as in closed-loop control. If the desired strategy will be practically feasible by the control system, particular quality specifications are to be met by the controlled object. They are the concept of observability and

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complementary concept of controllability. Recall, that in control theory, the importance of these concepts arises from the fact that they characterize mathematically the input-output structure of a system intended to be controlled [3]. This implies that the adequate mathematical model of the controlled object is to be developed.

The subject of the presented paper is to search for the ways for the most advantageous functional model of the process to be studied. Since the severe accident is too complicated to be formalized, the more advantageous heuristic model is to be constructed for our purpose. There isn't general approache for the structure construction of the mathematical model of the controlled object in uncertainty. As this is now realized in control theory as applied to multivariable nonlinear processes, if system components differ by their nature (physical, chemical and others), it is of highly importance to be able to describe their coherent behaviour [4]. Taking into consideration powerful algorithms of the theory of informational synthesis [5], we may say that information-theoretic approach is the governing factor for model construction.

The distinguishing feature, which is unique to nuclear accident gives us promise to improve the emergency control system efficiency. The evidence for this had been obtained during the work under the State Project of Belarus "Reliability and Safety" within 2003-2005 by the author. The subject matter of this investigation was to develop methodology of the retrospective analyses of radionuclide composition formation after severe accident. Present insights of severe accidents at the NPP from the standpoint of information theory will be discussed in the subsequent chapter.

2. Present-day insights of severe accident at the NPP

As it was noted above, dynamic instability is the key feature of the nuclear accident at NPP. Actually, parallel with the forces caused by non-linear feedback in the control system, there are other causes of chaotic motion, which may potentially be inherent to the severe accident dynamics[6]. However, the genuine nature of chaotic motion of the nuclear accident still remains to be unknown. Accident at the 4^{th} Unit of the Chernobyl NPP was the most sever accidents that had been happened over a fifty-year period of nuclear energy utilization. Let us discuss principal generalizations from its consequences.

There are not a lot of papers in which severe on-site radiation situation had been studied. Radiation situation at the close to epicenter areas was of extremely high levels of radiation fields (up to 100 R/h), irregular heterogeneous character of the radionuclide contamination. Fractal properties of the radiation fields within the bounds of 3-5 km around Chernobyl NPP just after the explosion had been studied in the work [7]. Nevertheless complicated dynamics that formed near-zone radiation fields just after the exposure is not understood clearly up to now.

As opposed to the epicenter area, radionuclide contamination in the outside regions was the subject of many studies. Atmospheric transport made radionuclides release to be available for measurement in the Ukraine, Russia and Belarus [8-12]. The main features of the site contamination resulting from atmospheric transport of the radionuclide release are spottiness, irregularity, multi-isotopic composition, time-variable characterization of the radiation fields.

The investigation that had been carried out in this direction in Belarus and, in particular by specialists of Joint Institute of Power and Nuclear Research (JIPNR BAS) is worth more attention to notice. Large-scale radiological measurement of the early contaminations was carried out in the Mogilev and Gomel regions since May, 1986. Then, the Data Bank of hazardous nuclide had been created at the Institute. They were the isotopes of ^{134}Cs, ^{137}Cs, ^{141}Ce, ^{144}Ce, ^{103}Ru, ^{106}Ru, ^{95}Zr, ^{140}Ba, ^{140}La. Although a large amount of the short-lived isotopes were not measured in the soil contamination in 1986, nevertheless the authors were able to calculate the short-lived radionuclide activities retrospectively and reconstruct radiation doses to population caused by the short-lived nuclides [11,12]. Let us list the most essential of them. There were the isotopes of ^{130}I ($T_{1/2}$=12,36h), ^{133}I($T_{1/2}$=20,8 h), ^{135}I ($T_{1/2}$=6,6h), ^{132}Te ($T_{1/2}$=78,2h) and its daughter nuclide ^{132}I ($T_{1/2}$=2,26h). The basic calculation technique that had been applied in this study consists in the theoretical modeling of the radionuclide composition formation in the accidental reactor, which is combined with the radioecology data from Data Bank. This procedure is generally referred to as isotope correlation technique. Summing up, we can say that the progress in reconstruction of the radionuclide contamination in Belarus together with the results of retrospective analysis that had been made in Russia and Ukraine may be consider as evidence that the system under study is completely observable. Followed by the conclusion that had been proved rigorously in paper [14], we may suppose that the system is completely controllable too. Let us assume that these statements are valid with regard to the internal variables describing radionuclide composition of the accidental process at least.

Radioactive decay chain is well known discrete Markov process in stochastic theory [13]. It is believed that owing to high correlation of its internal variables discrete Markov process is precisely the process that provides internal ordering of the system's dynamics. So the mathematical model of the radionuclide composition formation is the central of present research. We consider that the conceptions of information theory are the more adequate approach for the problem under investigation.

3. Information model of nuclear accident

3.1. Notion of information

Sever accident at the NPP is qualified as non-equilibrium process when the system modifies its macroscopic behaviour qualitatively. In other words, modifications touch upon its macroscopic space- and temporal structure. Insofar the exchange by energy, matter and information with the environment take place, it is well reasonable that nuclear accident can be described as macroscopic open system. Synergetic is the theory concerned with the structural complexity of the macroscopic open systems. At this point the information has become the instrument of study of the macroscopic open system's behaviour [15].

C. Shannon was the first who offered two definitions of information in his work on mathematical theory of communication [16]. One of information definitions is identical to that of Boltzmann's entropy and in fact gives a measure of statistical uncertainty of a process. Note, that this definition is known as S-information. It is in accordance with the quantitative measure of phenomena diversity given by W.R. Ashby [17] as well.

Let us define a measure of information $I(X)$ and uncertainty $H(X)$ (entropy) of discrete Markov source of information as follows. If there is a random discrete process X having the probability distribution function f, than the single function $H(X)$ is

$$H(X) = I(X) = -\sum_{n} f_i \log_2 f_i, \quad \sum_{n} f_i = 1. \tag{1}$$

It should be noted that since the S-information hasn't any meaning (semantics), then it is not sufficient for the analysis of open systems, in general, and for use in control systems in particular. At this point, more adequate conception of information had been done in terms of difference of unconditional S-entropy Boltzmann $H(X)$ and conditional entropy $H(X|Y)$ as follows:

$$I(X,Y) = H(X) - H(X|Y) \tag{2}$$

where $I(X,Y)$ denotes "correlation" information concerning with statistical correlation of the state function of the variables X and Y, and $H(X|Y)$ is expressed in terms of conditional probability distribution function $f\,(X|\,Y) = f(X|\,Y)\,f(Y)$. It should be emphasized the importance of the second notion of information for control system designing. This notion of information provides semantics and gives information measure in accord with the value of control parameters of open system. We will point out significant works of B.B. Kadomtsev and Yu. Klimontovich [18,19] along with the study of H. Haken in this context.

The third notion of information is a value of information. The most significant issue of this notion with respect to control system efficiency go back to Brilloun's negentropy and to the study of R.L. Stratonovich [20,21]. It is significant to note that for the control system to be capable to actuate automatically on controlled object in emergency, it must be able to analyse quality of information received according to a measure of its value.

3.2. The schematic model of information transfer system as applied to nuclear accident

Information theory concerns with the mathematical model of information transfer system and followed by the analysis of this model. We adopt Shannon's conception of information transfer system for developing informational model of a process under discussion. According to the standard schematic model of information transfer system given by C. Shannon (fig.1), the process starts from the discrete source that generates a set of signals (a message) into the communication path and of the information receiver.

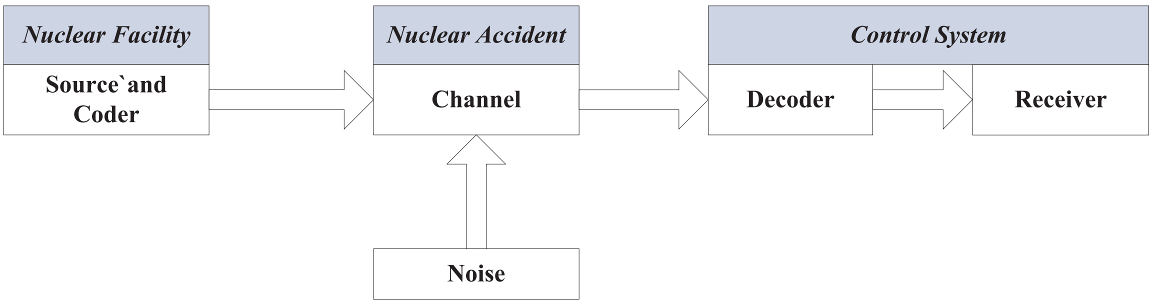

FIG. 1: Information Model of the Nuclear Accident at the NPP.

In his communication theory Shannon offered the constructive idea to use statistical properties of the source for information coding. As regards to radionuclide composition formation in case of nuclear accident, it seems very reasonable that discrete Markov process, related to radioactive decay chains is the best suited to mathematical model of a discrete source of information.

4. Mathematical model of radionuclide transmutation: CODE DECA

As the first step in this direction a code of the process of the information generation had been constructed. The author put forwards its own idea for developing mathematical model of radionuclide composition formation of the uranium fuel under the neutron irradiation in a nuclear facility [22]. Actually, the detailed network of all possible actinide transmutations and fission-product chains has extremely complex structure. It includes branching, feedback and cycle loops.To describe rigorously complex structure of the radionuclide formation network, we introduced into consideration the bond graph $G(V, E)$, where V is a set of a vertexes and E is a set of an edges, and each of $v_1, v_2, \ldots v_n$ vertexes is associated with the transmuting nuclide (fission product or actinide). Then a pair of the vertexes (v_i, v_j) forms the edge indicating radioactive transformation from i to j nuclide.

Let $|V|$ be a number of vertexes, then $|V|$ is equal to sum of $p = 650$ and $q = 58$ where p, q are quantity of fission products and actinides correspondingly. The bond structure of the oriented graph $G(V, E)$ is described by adjacency matrix $\hat{S}$ in such a way that matrix coefficient $s_{ij} = 1$ if and only if (v_i, v_j) belongs to the graph, $G(V, E)$ otherwise $s_{ij} = 0$. To describe fission product and actinide accumulation in uranium fuel irradiated by a neutron flux, the set of differential equations have been derived. The set of flows of matter on the graph $G(V, E)$ has been constructed in the following way. Let us denote x_i as concentration of i-radionuclide and a set of actinide and fission product concentrations by vector $\vec{x}(t) = [x_1, ..., x_p, x_{p+1}, ...x_{p+q}]$. Then we associated with the edge (i, j) of graph $G(V, E)$ the flow of i-radionuclide matter and denote it as $s_{ij}(t)\cdot\ x_i(t)$, where s_{ij} is a constant of formation j-nuclide from i- one. Then we can write the set of the ordinary differential equations described radionuclide transmutation in a form of

$$\frac{dx_i(t)}{dt} = -\sum_{j=1}^{m} s_{ij}(t)x_i(t) + \sum_{k=1}^{n} s_{ki}(t)x_k(t) \tag{3}$$

were i=1,708; m, n denote, correspondingly, a number of all possible receivers and sources of i–nuclide that belong to graph $G(V, E)$.

We wrote eq. (3) in vector form as

$$\frac{d\vec{x}(t)}{dt} = \hat{S}(t) \cdot \vec{x}(t) \tag{4}$$

with the time-dependent transmutation matrix $\hat{S}(t)$ and initial nuclide concentration $\vec{x}_o$ at the moment t_o. The dimension of matrix $\hat{S}$ is equal to $r = p + q = 708$.

Nuclear database on 650 fission products and onf 58 actinide nuclei has been constructed. It consists of : λ_i,- decay constants; y_i - fission product yields of ^{235}U, ^{239}Pu, ^{241}Pu; $\sigma(E)$ - neutron capture cross sections [23].

We consider here a reactor of RBMK-type. This type of reactor acts on thermal neutrons. The neutron-induced transmutation rates have been obtained with the use of code TRIFON and inserted here in a model in accordance to the formulas:

$$s_{ij}(t) = \int dV \int_0^\infty \sigma_{ij}(E)\Phi(\vec{R}, E, t)dE \tag{5}$$

$$s_{ij} = \sigma_{ij}^{av}\Phi_T \tag{6}$$

and

$$\Phi_T = \frac{w}{E_k^f \sum\limits_k (\sigma_k^f x_k^f)} \tag{7}$$

where $\Phi(\vec{R}, E, t)$ is energetic and spatial-distribution of the neutron density, w is specific power of the reactor, σ_{ij}^{av} *is* the energy-averaged cross section, Φ_T is the thermal-neutron flux, E_k^f is fission energy and x_k^f is concentration of fission nuclei of k-type (where k =1,2,3 and denotes U-235, Pu-239, Pu-241, correspondingly).

Suppose that for a time interval $\tau_j = t_j - t_{j-1}$ matrix $\hat{S}^j$ can be regarded as invariable. Then we can write formal solution of eq. (3) as the exponential function of matrix $\hat{S}^j$:

$$\vec{x}(t_j) = \exp[\hat{S}^j\tau_j]\vec{x}(t_0) = \hat{S}^j \cdot \vec{x}(t_0) \tag{8}$$

Then nuclide concentration $\vec{x}(t_n)$ and activity $\vec{A}_i(t_n)$ at the moment t_n can be calculated as follows:

$$\vec{x}(t_n) = B^n \cdot .. \cdot B^j \cdot .. \cdot B^1 \cdot \vec{x}(t_0) \tag{9}$$

$$\vec{A}_i(t) = \lambda_i \vec{x}_i(t) \tag{10}$$

In view of common problems of the system complexity, namely, large number of system dimension and the presence of highly dynamic processes flowing simultaneously with slowly variable ones, we used an improved numerical algorithm. To calculate numerically (8-9) we expand exponential function of matrix $\hat{S}$ into series with a small number K and short interval h by a formula

$$R(\hat{S}) = \sum_{k=1}^{K} \frac{(h\hat{S})^k}{k!} \tag{11}$$

. Then we obtain the solution at a moment $t = t_n$, as consistent with the following formula:

$$ed\vec{x}_n = \left(\sum_{k=1}^{K} \frac{\left(h\hat{S}\right)^k}{k!} \right)^n \cdot \vec{x}_0 \tag{12}$$

Calculation procedure expressed by (8-12), data of the nuclear database and of the $G(V,E)$ describing system's structure has been realised in code DECA.

To speed up the processing and economise computational resources the sparse matrix technology has been employed by author [24].

4.1. Numerical results

Numerical simulation of the nuclear fuel irradiation and accumulation of 58 actinide isotopes and of 650- fission product isotopes during the lifetime at RBMK-1000 with the use of code DECA were carried out. Accumulation of the long-lived radionuclides and short-lived one had been calculated. Then the activity correlations with respect to the nuclear fuel burn up had been studied.

The results had been obtained for the basic parameter of RBMK-1000. They are as follows: heat power is 3200 MWt, the load of uranium fuel with 2% of ^{235}U enrichment, fuel mass is equal to 190 tons, fuel life-time is 3 years.

The burn-up dependences of the isotopes plutonium, curium and americium reveals truly complex nature caused by the features of their transmutation chains. The burn-up behaviours of the isotopes ^{239}Pu, ^{240}Pu, ^{241}Pu, ^{236}U consentrations are represented in fig.2.

As to the long-lived fission products, their activities constantly increase as a fuel burn-up is enlarged. The results of the numerical calculation of the radionuclide activities of ^{134}Cs ($T_{1/2}$=2,06 y), ^{137}Cs ($T_{1/2}$=30,17 y) and ^{90}Sr ($T_{1/2}$=28,5y) depending on the fuel burn-up are shown in fig.3. The results of numerical calculations of the activities of the isotopes ^{131}I ($T_{1/2}$=8,04 d), ^{91}Sr ($T_{1/2}$=9,8h), ^{106}Ru ($T_{1/2}$=39,4d) depending on the fuel burn-up are shown in fig.4.

The short-lived isotopes have typically bell-shaped characteristic with maximum at approximately of 10 MWt·day/kg U. This fact indicates that when the equilibrium between the yield of radionuclide from fission and radioactive decay is being reached, then activity decreases as far as the specific power in the fuel assembly is reduced.

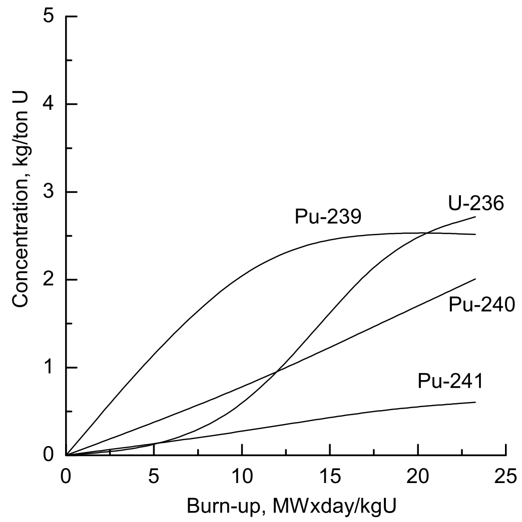

FIG. 2: Burn-up dependences of Pu-239, Pu-240, Pu-241, U-236 concentration.

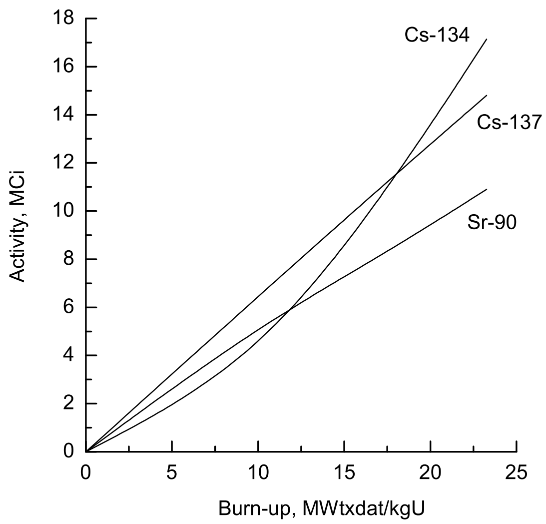

FIG. 3: Burn-up dependences of Sr-90, Cs-134, Cs-137 activities.

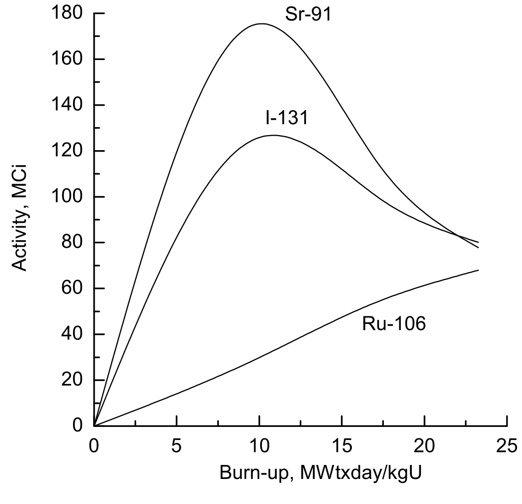

FIG. 4: Burn-up dependences Sr-91, Ru-103, I-131 activities.

As a whole the activities of the short-lived fission products strongly depend on the reactor power and their behaviour may indicate transient and accidental processes of the NPP. To illustrate this fact we calculated total activity burst initiated by an uncontrollable neutron kinetic in case of the accident at the Chernobyl NPP. Time dependences after the accident of the total activity decrease had been calculated for two initial conditions. Their behaviors are given in fig.5: a) starting at the moment after the uncontrolled power burst up to 320GWt and b) starting at the moment after minimum power operation of the 4^{th} Unit of ChNPP (P = 200 MWt) (before uncontrolled power burst).

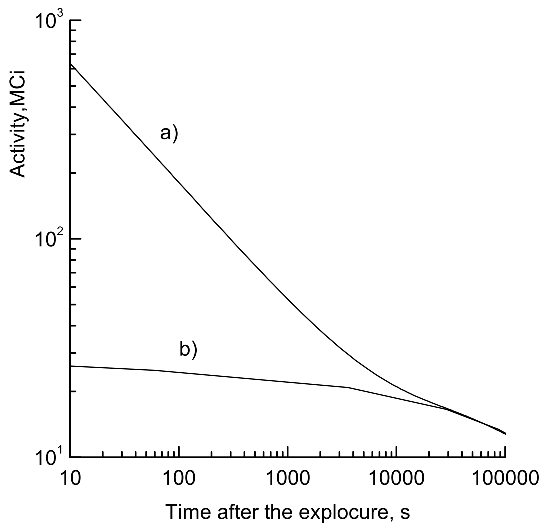

FIG. 5. Decrease of total nuclear fuel activity depending on the time after the accident: a) after uncontrolled neutron burst in the reactor up to 320GWt; b) before uncontrolled power burst (P = 200 MWt).

5. Concluding Remarks

The above cited papers [4,5,14] may be considered as the present-day information-theoretic base for the control system designing as applied to multivariable nonlinear processes. Unfortunately uncertain dynamics of the complex nonlinear process is the restrictive factor to achieve much success in its application. A unique feature of the severe accident that there was the subject of discussion in the presented paper, in contrast, would promote to find out the solution of the problem. Fission of the uranium fuel following the subsequent radioactive chains is discrete Markov source that generates information (a code) related to radionuclide composition in all stages of its formation: from nuclear fuel and during the accident as well. The results of the paper [11] had been demonstrated significant indications of real accidental conditions by example of Chernobyl accident. We consider that information model of the nuclear accident developed in the presented paper is the most advantageous model for quantitative characterization in terms of information measuring. At our glance this approach may bring new perspectives for developing the advantageous technique for emergency control system that is known as recognition method. This study is the first level of investigation in this direction. It is necessary much broader research project in the fields of information theory and nonlinear dynamics to cope a problem of emergency control system in case of nuclear accident.

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