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On products of arbitrary number of Mueller matrices

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The general expressions for the products of arbitrary number of some known simple Mueller matrices are obtained. It is proposed and used a general approach based on introduction of special basis in space of the Stokes vector parameters and writing Mueller matrices in appropriate block form.

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It is known (see [1-3]) that for description of the energetical and polarization characteristics of the optical radiation the four-dimensional Stokes vector parameters

$$\tilde{S} = \{I, P_1, P_2, P_3\}$$

are used. The parameter I characterizes here the complete intensity of the radiation, P_1 - predominantly horizontal polarization, P_2 - predominantly polarization at an angle of 45^0, P_3 - predominantly right-circular polarization.

The optical characteristics of the device (analyzer) which transforms the radiation are usually described with help of the Mueller matrices connecting the Stokes parameters before and after the passage of the light through the device.

In the simplest, so called ideal case the Mueller matrix characterizing an analyzer with horizontal passage axis has the following form [1-3]:

$$M^0 = M(0) = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{1}$$

In determining the Stokes parameters and Mueller matrices the space orientation of the device is to be taken into account and the corresponding transition transformations from an initial reference frame to another one are to be invoked.

Particularly, in the case when reference frame is rotated through the angle ϑ about the light propagation direction the rotation matrix is defined in the form [1-3]

$$R = R(\vartheta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & C_2 & S_2 & 0 \\ 0 & -S_2 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad R^{-1}(\vartheta) = R(-\vartheta) = \tilde{R}(\vartheta), \tag{2}$$

where

$$C_2 = \cos 2\vartheta, \qquad S_2 = \sin 2\vartheta. \tag{3}$$

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The Mueller matrix (1) under acting of the rotation matrix (2) is transformed as follows:

$$M = M(\vartheta) = R(-\vartheta)M(0)R(\vartheta) = \frac{1}{2}\begin{pmatrix} 1 & C_2 & S_2 & 0 \\ C_2 & C_2^2 & C_2S_2 & 0 \\ S_2 & S_2C_2 & S_2^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (4)$$

In following, it is convenient to pass to the special basis in the space of the Stokes vector parameters, by introducing the following transformation (permutation) matrix:

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \qquad P^{-1} = \tilde{P}.$$

After transition to the new basis and to the transformed Stokes vector $\tilde{S}' = \tilde{PS} = \{I, P_2, P_3, P_1\}$, the 44 matrices $M(0)$ (1), $R(\vartheta)$ (2) $M(\vartheta)$ (4) take the easy-to calculations block form

$$M^{0'} = M'(0) = \tilde{P}M(0)P = \frac{1}{2}\begin{pmatrix} \beta^0 & \beta^0 \\ \beta^0 & \beta^0 \end{pmatrix}, \qquad (5)$$

$$R' = R'(\vartheta) = \tilde{P}R(\vartheta)P = \begin{pmatrix} I_2 & 0 \\ 0 & O(\vartheta) \end{pmatrix}, \qquad R^{-1}(\vartheta) = \begin{pmatrix} I_2 & 0 \\ 0 & \tilde{O}(\vartheta) \end{pmatrix}, \qquad (6)$$

$$M' = M'(\vartheta) = \tilde{P}M(\vartheta)P = \frac{1}{2}\begin{pmatrix} \beta^0 & \beta \\ \tilde{\beta} & \tilde{\beta}\beta \end{pmatrix}. \qquad (7)$$

Here the following notations for the related block 22-matrices are introduced (see (3)):

$$O = O(\vartheta) = \begin{pmatrix} C_2 & S_2 \\ -S_2 & C_2 \end{pmatrix}, \qquad O^{-1} = O(-\vartheta) = \tilde{O}(\vartheta) = \begin{pmatrix} C_2 & -S_2 \\ S_2 & C_2 \end{pmatrix}; \qquad (8)$$

$$\beta^0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \beta = O\beta^0 = \begin{pmatrix} C_2 & S_2 \\ 0 & 0 \end{pmatrix}, \qquad \tilde{\beta} = \beta^0\tilde{O} = \begin{pmatrix} C_2 & 0 \\ S_2 & 0 \end{pmatrix}. \qquad (9)$$

Now let us consider the problem of construction of the resulting Mueller matrix M which corresponds to the action of a series (product) of arbitrary number n of different matrice M_i

$$M = M_n M_{n-1} \ldots M_i \ldots M_2 M_1,$$

taken in the form (7)

$$M_i = M(\vartheta_i) = \frac{1}{2}\begin{pmatrix} \beta^0 & \beta_i \\ \tilde{\beta}_i & \tilde{\beta}_i\beta_i \end{pmatrix}, \qquad \beta_i = \begin{pmatrix} C_2^{(i)} & S_2^{(i)} \\ 0 & 0 \end{pmatrix}, \qquad \tilde{\beta}_i = \begin{pmatrix} C_2^{(i)} & 0 \\ S_2^{(i)} & 0 \end{pmatrix}. \qquad (10)$$

In the simplest case of the product of two such Mueller matrices $M_2 = M(\vartheta_2)$ $M_1 = M(\vartheta_1)$ (10) after elementary calculations we will have

$$M_2M_1 = M(\vartheta_2)M(\vartheta_1) = \frac{1}{2}\begin{pmatrix} \beta^0 & \beta_2 \\ \tilde{\beta}_2 & \tilde{\beta}_2\beta_2 \end{pmatrix}\frac{1}{2}\begin{pmatrix} \beta^0 & \beta_1 \\ \tilde{\beta}_1 & \tilde{\beta}_1\beta_1 \end{pmatrix} =$$

$$= \frac{1}{4}\begin{pmatrix} \beta^0 + \beta_2\tilde{\beta}_1 & \beta_1 + \beta_2\tilde{\beta}_1\beta_1 \\ \tilde{\beta}_2 + \tilde{\beta}_2\beta_2\tilde{\beta}_1 & \tilde{\beta}_2\beta_1 + \tilde{\beta}_2\beta_2\tilde{\beta}_1\beta_1 \end{pmatrix} = C^2_{(\vartheta_2-\vartheta_1)}\frac{1}{2}\begin{pmatrix} \beta^0 & \beta_1 \\ \tilde{\beta}_2 & \tilde{\beta}_2\beta_1 \end{pmatrix}, \qquad (11)$$

where is taken into account that (see (3), (9))

$$\begin{gathered}(\beta^0)^2 = \beta^0, \qquad \beta^0\beta_1 = \beta_1, \qquad \tilde{\beta}_2\beta^0 = \tilde{\beta}_2, \qquad \beta_2\tilde{\beta}_1 = C_{2(\vartheta_2-\vartheta_1)}\beta^0, \\ (1+\beta_2\tilde{\beta}_1)\beta^0 = (1 + C_2''C_2' + S_2''S_2')\beta^0 = (1 + C_{2(\vartheta_2-\vartheta_1)})\beta^0 = 2C^2_{(\vartheta_2-\vartheta_1)}\beta^0;\end{gathered} \tag{12}$$

$$\begin{gathered}C_2'' = \cos(2\vartheta_2), \qquad S_2'' = \sin(2\vartheta_2), \qquad C_2' = \cos(2\vartheta_1), \qquad S_2' = \sin(2\vartheta_1), \\ C_{2(\vartheta_2-\vartheta_1)} = \cos[2(\vartheta_2-\vartheta_1)], \qquad C_{(\vartheta_2-\vartheta_1)} = \cos(\vartheta_2-\vartheta_1).\end{gathered} \tag{13}$$

In similar manner we can also find the product of three matrices

$$M_3M_2M_1 = M(\vartheta_3)M(\vartheta_2)M(\vartheta_1) = C^2_{(\vartheta_3-\vartheta_2)}C^2_{(\vartheta_2-\vartheta_1)}\frac{1}{2}\begin{pmatrix}\beta^0 & \beta_1\\ \tilde{\beta}_3 & \tilde{\beta}_3\beta_1\end{pmatrix}. \tag{14}$$

It immediately follows the general expression for the product of n Mueller matrices $M_i = M_i(\vartheta_i)$ (10):

$$\begin{gathered}M = M_nM_{n-1}\ ...\ M_i\ ...\ M_2M_1 = M(\vartheta_n)M(\vartheta_{n-1})\ ...\ M(\vartheta_i)\ ...\ M(\vartheta_2)M(\vartheta_1) = \\ = C^2_{(\vartheta_n-\vartheta_{n-1})}C^2_{(\vartheta_{n-1}-\vartheta_{n-2})}\ ...\ C^2_{(\vartheta_i-\vartheta_{i-1})}\ ...\ C^2_{(\vartheta_2-\vartheta_1)}\frac{1}{2}\begin{pmatrix}\beta^0 & \beta_1\\ \tilde{\beta}_n & \tilde{\beta}_n\beta_1\end{pmatrix}.\end{gathered} \tag{15}$$

Since the Mueller matrices under consideration $M(0)$ (5), as well as $M(\vartheta)$ (10) are idempotent $M^2(0) = M(0)$, $M^2(\vartheta) = M(\vartheta)$, the action of the product of n such identical matrices is equivalent to the action of the one matrix alone. —

Now let us pass to the consideration of the non-ideal situation, particularly to the case of the known Mueller matrix

$$M^0 = \bar{\tau}\begin{pmatrix}1 & 0 & q & 0\\ 0 & \sqrt{1-q^2} & 0 & 0\\ q & 0 & 1 & 0\\ 0 & 0 & 0 & \sqrt{1-q^2}\end{pmatrix} = \tilde{\tau}\begin{pmatrix}\alpha & \beta^0 q\\ \beta^0 q & \alpha\end{pmatrix}, \tag{17}$$

which characterizes an analyzer with polarizing capacity q_i and average passage coefficient $\bar{\tau}_i$. The block 2x2-matrices in (17) are defined as (see (3),(10))

$$\begin{pmatrix}1 & 0\\ 0 & \sqrt{1-q^2}\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & g\end{pmatrix} = \alpha, \qquad g = \sqrt{1-q^2}, \qquad \begin{pmatrix}q & 0\\ 0 & 0\end{pmatrix} = \beta^0 q, \tag{18}$$

where

$$\alpha\beta^0 = \beta^0\alpha = \beta^0. \tag{19}$$

As above, let us to consider a problem of construction of of general expressions for products

$$M^0 = M^0_nM^0_{n-1}\ ...\ M^0_i\ ...\ M^0_2M^0_1 = M^0_{n,n-1,...,i,...,2,1},$$

of the arbitrary number n of the Mueller matrices M^0_i taken in the form (see (17),(18),(19))

$$M^0_i = \bar{\tau}_i\begin{pmatrix}\alpha_i & \beta^0 q_i\\ \beta^0 q_i & \alpha_i\end{pmatrix}, \qquad \alpha_i = \begin{pmatrix}1 & 0\\ 0 & g_i\end{pmatrix}, \qquad g_i = \sqrt{1-q_i^2}. \tag{20}$$

First of all, we find the explicit expressions for the products of two and three matrices M^0_i (20)

$$M^0_{2,1} = M^0_2M^0_1 = \bar{\tau}_2\bar{\tau}_1\begin{pmatrix}\alpha_2\alpha_1 + \beta^0 M(q_2,q_1) & \beta^0 N(q_2,q_1)\\ \beta^0 N(q_2,q_1) & \alpha_2\alpha_1 + \beta^0 M(q_2,q_1)\end{pmatrix}, \tag{21}$$

$$M(q_2,q_1)=q_2q_1=\sum_{i>j}^{C_2^1=1}q_iq_j,\qquad N(q_2,q_1)=q_2+q_1=\sum_{i}^{C_2^1=2}q_i;$$

$$M^0_{3,2,1}=M^0_3M^0_2M^0_1=\bar\tau_3\bar\tau_2\bar\tau_1\begin{pmatrix}\alpha_3\alpha_2\alpha_1+\beta^0M(q_3,q_2,q_1) & \beta^0N(q_3,q_2,q_1)\\ \beta^0N(q_3,q_2,q_1) & \alpha_3\alpha_2\alpha_1+\beta^0M(q_3,q_2,q_1)\end{pmatrix},\tag{22}$$

$$M(q_3,q_2,q_1)=\sum_{i>j}^{C_3^2=3}q_iq_j,\qquad N(q_3,q_2,q_1)=\sum_{i}^{C_3^1=3}q_i+\sum_{i>j>k}^{C_3^3=1}q_iq_jq_k.$$

Hence, it is easy to conclude that the general expression for the product of arbitrary number n of the different Mueller matrices N_i^0 (20) may be written in the following compact form

$$M^0_{n,n-1,...,i,...,2,1}=M^0_nM^0_{n-1}\,...\,M^0_i\,...\,M^0_2M^0_1=$$

$$=\bar\tau_n\bar\tau_{n-1}...\bar\tau_i...\bar\tau_2\bar\tau_1\begin{pmatrix}A_{n,n-1,...,i,...,2,1} & B_{n,n-1,...,i,...,2,1}\\ B_{n,n-1,...,i,...,2,1} & A_{n,n-1,...,i,...,2,1}\end{pmatrix},\tag{23}$$

$$A_{n,n-1,...,i,...,2,1}=\alpha_n\alpha_{n-1}...\alpha_i...\alpha_2\alpha_1+\beta^0M(q_n,q_{n-1},...,q_i,...,q_2q_1),$$

$$M(q_n,q_{n-1},...,q_i,...,q_2,q_1)=\sum_{i>j}^{C_n^2}q_iq_j+\sum_{i>j>k>l}^{C_n^4}q_iq_jq_kq_l+...,$$

$$B_{n,n-1,...,i,...,2,1}=\beta^0N(q_n,q_{n-1},...,q_i,...,q_2q_1),$$

$$N(q_n,q_{n-1},...,q_i,...,q_2q_1)=\sum_{i}^{C_n^1}q_i+\sum_{i>j>k}^{C_n^3}q_iq_jq_k+....$$

By putting

$$\bar\tau_i=\bar\tau,\qquad q_i=q,\qquad \alpha_i=\alpha,\tag{24}$$

i.e. by passing to the case of identical Mueller matrices $M_i^0(q_i)=M^0(q)$ (20) we obtain more simple expressions for products of 2, 3 and n matrices under consideration:

$$(M^0)^2=(\bar\tau)^2\begin{pmatrix}\alpha^2+\beta^0q^2 & \beta^0(2q)\\ \beta^0(2q) & \alpha^2+\beta^0q^2\end{pmatrix},\tag{25}$$

$$(M^0)^3=(\bar\tau)^3\begin{pmatrix}\alpha^3+\beta^0(3q^2) & \beta^0(3q+q^3)\\ \beta^0(3q+q^3) & \alpha^3+\beta^0(3q^2)\end{pmatrix}\tag{26}$$

$$(M^0)^n=(\bar\tau)^n\begin{pmatrix}\alpha^n+\beta^0(C_n^2q^2+C_n^4q^4+...) & \beta^0(C_n^1q+C_n^3q^3+...)\\ \beta^0(C_n^1q+C_n^3q^3+...) & \alpha^n+\beta^0(C_n^2q^2+C_n^4q^4+...)\end{pmatrix}.\tag{27}$$

The situation turns out to be more complicate in the case when the initial Mueller matrices $M_i^0=M_i(0)$ (20) are transformed with help of the rotation matrices $R_i=R(\vartheta_i)$ (6) and, as a result, take the form:

$$M_i=M_i(\vartheta_i)=R^{-1}(\vartheta_i)M_i(0)R(\vartheta_i)=$$

$$=\bar\tau\begin{pmatrix}I_2 & 0\\ 0 & \tilde O(\vartheta_i)\end{pmatrix}\begin{pmatrix}\alpha_i & \beta^0q_i\\ \beta^0q_i & \alpha_i\end{pmatrix}\begin{pmatrix}I_2 & 0\\ 0 & O(\vartheta_i)\end{pmatrix}=\tilde\tau\begin{pmatrix}\alpha_i & \beta_iq_i\\ \tilde\beta_iq_i & \gamma_i\end{pmatrix},\tag{28}$$

where the additional notations are introduced (see (3), (9), (18), (19))

$$\gamma_i=\tilde O\alpha_iO=\begin{pmatrix}C_2^{(i)2} & C_2^{(i)}S_2^{(i)}\\ C_2^{(i)}S_2^{(i)} & S_2^{(i)2}\end{pmatrix}+\begin{pmatrix}S_2^{(i)2} & -S_2^{(i)}C_2^{(i)}\\ -C_2^{(i)}S_2^{(i)} & C_2^{(i)2}\end{pmatrix}g_i=\tilde\beta_i\beta_i+\tilde\delta_i\delta_ig_i,\tag{29}$$

$$\delta_i = \begin{pmatrix} -S_2^{(i)} & C_2^{(i)} \\ 0 & 0 \end{pmatrix}, \quad \tilde{\delta}_I = \begin{pmatrix} -S_2^{(i)} & 0 \\ C_2^{(i)} & 0 \end{pmatrix}.$$

For simplicity, we shall restrict our consideration to the case when the all initial different Mueller matrices $M_i^0 = M_i(0)$ (28) are transformed with help of the one and the same rotation matrix $R(\vartheta)$ (6).

As a result, for the product of n transformed different Mueller matrices(28)(see(29))

$$M_i = M_i(\vartheta) = R(-\vartheta)M_i^0 R(\vartheta) = \tilde{\tau}_l \begin{pmatrix} \alpha_l & \beta q_i \\ \tilde{\beta} q_i & \gamma_i \end{pmatrix}, \quad \gamma_i = \tilde{\beta}\beta + \tilde{\delta}\delta g_i, \tag{30}$$

we can, by taking into account that, by definition, $R^{-1}R = R(-\vartheta)R(\vartheta) = R(\vartheta)R(-\vartheta) = RR^{-1} = I$, write

$$M_n(\vartheta)M_{n-1}(\vartheta) \ldots M_i(\vartheta) \ldots M_2(\vartheta)M_1(\vartheta) =$$

$$= R(-\vartheta)(M_n^0 M_{n-1}^0 \ldots M_i^0 \ldots M_2^0 M_1^0)R(\vartheta), \tag{31}$$

where the expression under transformation, $M_n^0 M_{n-1}^0 \ldots M_i^0 \ldots M_2^0 M_1^0$, is defined by the above obtained general formula (23).

After corresponding calculations for the products of 2,3 and n Mueller matrices (30) we find

$$M_2 M_1 = M_2(\vartheta)M_1(\vartheta) = R(-\vartheta)M_2^0 M_1^0 R(\vartheta) =$$

$$= R(-\vartheta)\left\{\bar{\tau}_2\bar{\tau}_1 \begin{pmatrix} \alpha_2\alpha_1 + \beta^0 q_2 q_1 & \beta^0(q_2+q_1) \\ \beta^0(q_2+q_1) & \alpha_2\alpha_1 + \beta^0 q_2 q_1 \end{pmatrix}\right\} R(\vartheta) = \tag{32}$$

$$= \bar{\tau}_2\bar{\tau}_1 \begin{pmatrix} \alpha_2\alpha_1 + \beta^0 q_2 q_1 & \beta(q_2+q_1) \\ \tilde{\beta}(q_2+q_1) & \tilde{\beta}\beta(1+q_2q_1) + \tilde{\delta}\delta g_2 g_1 \end{pmatrix},$$

$$M_3M_2M_1 = M_3(\vartheta)M_2(\vartheta)M_1(\vartheta) = R(-\vartheta)M_3^0 M_2^0 M_1^0 R(\vartheta) =$$

$$= \bar{\tau}_3\bar{\tau}_2\bar{\tau}_1 \begin{pmatrix} \alpha_3\alpha_2\alpha_1 + \beta^0 M(q_3q_2q_1) & \beta N(q_3,q_2,q_1) \\ \tilde{\beta}N(q_3,q_2,q_1) & \tilde{\beta}\beta[1 + M(q_3,q_2,q_1)] + \tilde{\delta}\delta g_3g_2g_1 \end{pmatrix}, \tag{33}$$

$$M(q_3,q_2,q_1) = q_3q_2 + q_3q_1 + q_2q_1, \qquad N(q_3,q_2,q_1) = q_3 + q_2 + q_1 + q_3q_2q_1.$$

$$M_n(\vartheta)M_{n-1}(\vartheta) \ldots M_i(\vartheta) \ldots M_2(\vartheta)M_1(\vartheta) =$$

$$= \bar{\tau}_n\bar{\tau}_{(n-1)}\ldots\bar{\tau}_i\ldots\bar{\tau}_2\bar{\tau}_1 \begin{pmatrix} A_{n,n-1,\ldots,i,\ldots,2,1} & B'_{n,n-1,\ldots,i,\ldots,2,1} \\ \tilde{B}'_{n,n-1,\ldots,i,\ldots,2,1} & A'_{n,n-1,\ldots,i,\ldots,2,1} \end{pmatrix},$$

$$B'_{n,n-1,\ldots,i,\ldots,2,1} = \beta N(q_n, q_{n-1}, \ldots, q_i, \ldots, q_2, q_1), \tag{34}$$

$$\tilde{B}'_{n,n-1,\ldots,i,\ldots,2,1} = \tilde{\beta} N(q_n, q_{n-1}, \ldots, q_i, \ldots, q_2, q_1),$$

$$A'_{n,n-1,\ldots,i,\ldots,2,1} = \tilde{O}(\vartheta)A_{n,n-1,\ldots,i,\ldots,2,1}O(\vartheta) =$$

$$= \tilde{\beta}\beta[1 + M(q_n, q_{n-1}, \ldots, q_i, \ldots, q_2, q_1)] +$$

$$+ \tilde{\delta}\delta g_n g_{n-1}\ldots g_i \ldots g_2 g_1.$$

In more simple case of products of identical Mueller matrices(30), transformed with help of the one and the same rotation matrix $R(\vartheta)$ (6), in correspondence with (32), (33), (34) and (25), (26), (27), we can immediately write

$$[M(\vartheta)]^2 = (\bar{\tau})^2 \begin{pmatrix} \alpha^2 + \beta^0 q^2 & \beta(2q) \\ \tilde{\beta}(2q) & \tilde{\beta}\beta(1+q^2) + \tilde{\delta}\delta g^2 \end{pmatrix}, \tag{35}$$

$$[M(\vartheta)]^3 = (\bar{\tau})^3 \begin{pmatrix} \alpha^3 + \beta^0(3q^2) & \beta(3q+q^3) \\ \tilde{\beta}(3q+q^3) & \tilde{\beta}\beta(1+3q^2) + \tilde{\delta}\delta g^3 \end{pmatrix}, \tag{36}$$

$$[M(\vartheta)]^n = (\bar{\tau})^n \begin{pmatrix} \alpha^n + \beta^0(C_n^2 q^2 + C_n^4 q^4 + ...) & \beta(C_n^1 q + C_n^3 q^3 + ...) \\ \tilde{\beta}(C_n^1 q + C_n^3 q^3 + ...) & \tilde{\beta}\beta(1 + C_n^2 q^2 + C_n^4 q^4 + ...) + \tilde{\delta}\delta g^n \end{pmatrix}. \tag{37}$$

Finally, let us consider a limiting case when polarizing capacity of the device q_i is very small and we can put $g_i = \sqrt{1-q_i^2} \to 1$.

Then the Mueller matrix (30) in this approximation takes the form ($\alpha_i \to I_2, \gamma_i \to I_2$)

$$M_i = M(q_i, \vartheta_i) = \bar{\tau}_l \begin{pmatrix} I_2 & \beta_i q_i \\ \tilde{\beta}_i q_i & I_2 \end{pmatrix}, \tag{38}$$

and for the product of arbitrary number of different matrices (40) transformed with help of different rotation matrices (6) in the first order approximation with respect to the small parameters q_i we can obtain the following simple expression:

$$M_n(\vartheta_n) M_{n-1}(\vartheta_n - 1) \ ... \ M_i(\vartheta_i) \ ... \ M_2(\vartheta_2) M_1(\vartheta_1) =$$

$$= \bar{\tau}_n \bar{\tau}_{n-1} ... \bar{\tau}_i ... \bar{\tau}_2 \bar{\tau}_1 \begin{pmatrix} I_2 & \sum_i^n \beta_i q_i \\ \sum_i^n \tilde{\beta}_i q_i & I_2 \end{pmatrix}. \tag{39}$$

Here, naturally, the all products of small parameters q_i are neglected.

Particularly, in the case when the all Mueller matrices (38) in the product (39) are obtained with help of the one and the same rotation matrix (6) (see (24)) we will have

$$M_n(\vartheta) M_{n-1}(\vartheta) \ ... \ M_i(\vartheta) \ ... \ M_2(\vartheta) M_1(\vartheta) =$$

$$= \bar{\tau}_n \bar{\tau}_{n-1} ... \bar{\tau}_i ... \bar{\tau}_2 \bar{\tau}_1 \begin{pmatrix} I_2 & \beta \sum_i^n q_i \\ \tilde{\beta} \sum_i^n q_i & I_2 \end{pmatrix}. \tag{40}$$

Thus, the action of the product (40) of n Mueller matrices (38) taken in the approximation under consideration turns out to be equivalent (up to products of the passage coefficients $\bar{\tau}_i$) to the action of the one separate matrix (38) alone with polarizing capacity equal to the $\sum_i^n q_i$ i.e, to the sum of capacities q_i of the all Mueller matrices in (40).

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