Nonlinear Dynamics and Applications. Vol. 14 (2007) 250 - 261

Accelerating Universe and Pioneer Anomaly as effects of the conformal deformation of time

L. M. Tomilchik*
Stepanov Institute of physics of NAS of Belarus
Nezalezhnasti Av., 68, Minsk, 220072, Belarus

It is shown that the phenomenon of the Universe acceleration and the Pioneer Anomaly can be explained as a purely kinematic consequences of the $SO(4,2)$ conformal group transformations treated as the coordinate transformations from the local Lorentz reference frame to the uniformly accelerating one.

It is shown that special conformal transformations, which preserving the light cone equation, nevertheless brings into existence the nonlinear transformation of its generating lines (conformal deformation of time or the time inhomogeneity). The explicit expression defining the location cosmological distance $R(z)$ and for the ratio $\phi(z) = V(z)/R(z)$ are obtained. It reproduce in the limit $z \to 0$ the conventional form of the Hubble law. The connection between acceleration and the Hubble constant $w = \frac{1}{2}cH_0$ follows herefrom immediately.

The expression describing the conformal time deformation in the small time limit predict the existence of the uniformly changing blue-shifted frequency drift. Its magnitude $\Delta\nu_{obs}$ is connected with the Hubble constant according to $\Delta\nu_{obs} = \nu_0 H_0 t$ (where ν_0 is the frequency of the signal emitted, t is the time of the signal propagation from the emitter to the receiver). The obtained formulae reproduce the PA experimental data. The function $\phi(z) = V(z)/R(z)$ generalizing the conventional expression for the Hubble law possesses a maximum at the point $z_0 \simeq 0.475$. That, in fact, reproduce the experimentally observed phenomenon which in the frame of the conventional cosmological paradigm is treated as the transition from the decelerated expansion of the Universe to the accelerated one.

PACS numbers: 02.20.Hj, 04.20.Cv
Keywords: Universe acceleration, Pioneer Anomaly, conformal transformations, time inhomogeneity

1. Introduction

The anomalous blue-shifted frequency drift detected in the microwave signals retransmitted by satellites Pioneer 10/11 and other spaceships (Pioneer Anomaly — PA) as yet remains unexplained (see [1–4]). The PA phenomenon is interpreted as an additional blue Doppler shift arising as a consequence of the existence of an anomalous unmodelled additional acceleration apprximately sunward directed and having the magnitude a_P equal to $a_P = (8.74 \pm 1.33) \cdot 10^{-8}$ cm/s^2. The intriguing proximity of a_P to the product cH_0 (c is the speed of light, H_0 is the Hubble constant) was noticed by several authors (see [2]). This suggests that PA can be treated as a local manifestation of the cosmological expansion. But a certain unambiguous theoretical arguments in favour of such a hypothesis are absent up to now. In this report we show that the phenomenon of the Universe acceleration discovered in 1998 [5,6] and the PA can be explained as a purely kinematic consequences of the $SO(4,2)$ conformal group transformations treated as the coordinate transformations from the local Lorentz reference frame to the uniformly accelerating

*E-mail: `lmt@dragon.bas-net.by`

one.

2. Conformal transformations and the accelerated frame of reference

It is well known that the special conformal transformations (SCT)

$$x'^{\mu} = \frac{x^{\mu} + a^{\mu}(x^{\alpha}x_{\alpha})}{1 + 2(a^{\alpha}x_{\alpha}) + (a^{\alpha}a_{\alpha})(x^{\beta}x_{\beta})}, \tag{1}$$

where a^{μ} is the four-vector parameter, $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$, can be interpreted as the transformations between the Lorentz (inertial) frame of reference $S(x^{\mu})$ and the noninertial (accelerated) frame of reference $S'(x'^{\mu})$ (see, for example, [7]). Following [7] we shall for simplicity consider a two-dimensional subspace $\{t, x\}$, i.e. we put

$$x^{\mu} = (ct, x, 0, 0), \;\; a^{\mu} = (0, -\frac{w}{2c^2}, 0, 0), \tag{2}$$

where w is a constant, having the dimension of acceleration.

Let us write the transformations (1) in the following noncovariant form:

$$\left.\begin{aligned} x' &= \frac{x\left(1+\frac{wx}{2c^2}\right) - \frac{wt^2}{2}}{\left(1+\frac{wx}{2c^2}\right)^2 - \left(\frac{wt}{2c}\right)^2}, \\ t' &= \frac{t}{\left(1+\frac{wx}{2c^2}\right)^2 - \left(\frac{wt}{2c}\right)^2}. \end{aligned}\right\} \tag{3}$$

In the case when $\frac{wx}{2c^2}$ and $\frac{wt}{2c}$ are negligible from (3) we have the formulae

$$x' = x - \frac{wt^2}{2}, \qquad t' = t, \tag{4}$$

which corresponds to Galilei-Newton kinematics. It should be noted that the identification of w with a constant three-dimensional newtonian acceleration is essentially based on this correspondence. It is clear from (3) also that sign $(-)$ of the vector a^{μ} x-component describes positive direction of S' acceleration along x-axe of inertial reference frame (IRF) S.

in fact, parameter a^{μ} of the form (2) and the 4-vector x^{μ} are defined in the comoving reference frame (CRF).

It is well known that world lines $\{x_0, x_{fix}\}$, where x_{fix} is fixed, in the (x_0, x) plane under the transformations (3) converts in hyperbolas in the (x_0', x') plane. Two such lines are showed on Fig. 1.

The meaning of the transformation is clear: every point fixed in Lorentz RF S is moving hyperbolically in non-inertial RF S'.

The transformation of coordinate differentials is convenient to express by (3) in the following symmetric form:

$$\left.\begin{aligned} dx' &= \frac{(\xi^2+\eta^2)dx - 2\xi\eta dx_0}{\xi^2-\eta^2}, \\ dx_0' &= \frac{(\xi^2+\eta^2)dx_0 - 2\xi\eta dx}{\xi^2-\eta^2}. \end{aligned}\right\} \tag{5}$$

Here

$$\xi = 1 + \frac{x}{2r_0}, \quad \eta = \frac{x}{2r_0}, \quad x_0 = ct, \quad x_0' = ct', \quad r_0 = \frac{c^2}{w}.$$

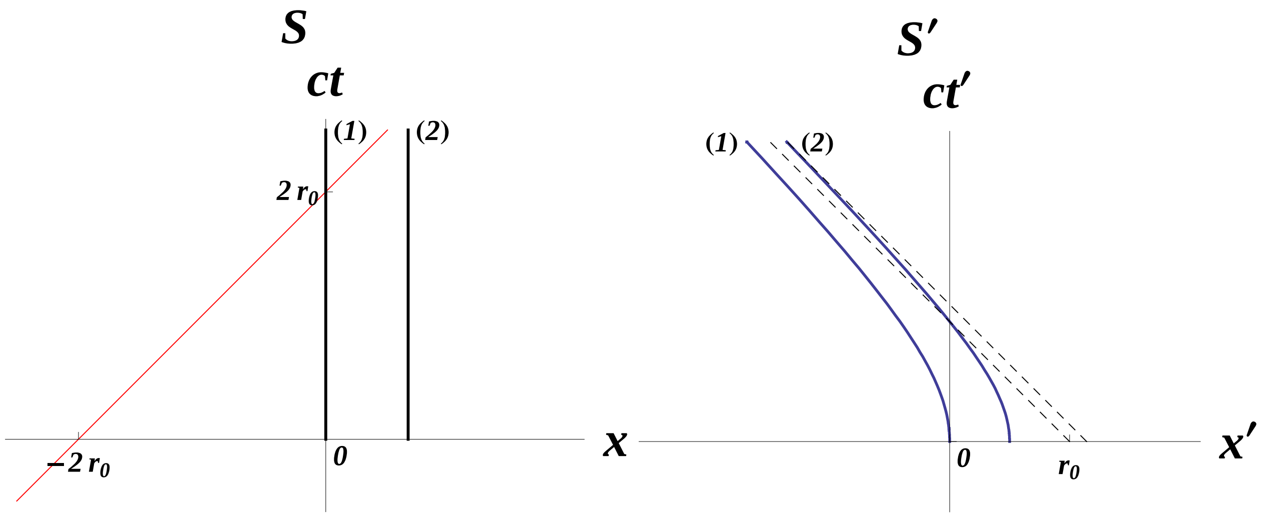

Figure 1. World lines of fixed particles (1) and (2) transforms into hyperbolas corresponding $\alpha = 1$ and $\frac{4}{3}$. Asymptotes of the hyperbolas are the dashed lines. For the sake of simplicity only positive time values are shown.

This results in the following expression for velocity transformations $V'_x = \dfrac{dx'}{dt'}$, $V_x = \dfrac{dx}{dt}$

$$V'_x = \frac{V_x - V_w}{1 - \frac{V_x V_w}{c^2}}, \tag{6}$$

where the "effective relative velocity" of S and S' RF V_w is

$$V_w = c\frac{2\xi\eta}{\xi^2 + \eta^2} \leqslant c.$$

The value of V_w is a function of space and time coordinates. It is notable that form of (6) is exactly the same as of usual velocity transformations in Special Relativity. In the non-relativistic approximation $\xi \approx 1,\ \eta \ll 1$ we receive

$$V'_x = V_x - wt, \tag{7}$$

corresponding to Galilei-Newton kinematics. The maximum of V_w is determined by $|\xi| = |\eta|$ and is equal to c.

From (7) the existence of blue Doppler drift immediately follows. Really due to (7) the longitudinal component of a point velocity V'_x measured by an observer fixed in non-inertial RF S' will be less then that measured by an observer fixed in the inertial comoving RF S by $\Delta V_x = wt$.

Clearly in this case an observer with fixed space coordinates in RF S' measures blue frequency shift as compared to an observer fixed in S. In fact, measured frequency of the signal of a moving away emitter in the non-relativistic limit will be

$$\nu' = \nu_0 \left(1 - \frac{V'_x}{c}\right) \qquad \text{in RF } S',$$
$$\nu' = \nu_0 \left(1 - \frac{V_x}{c}\right) = \nu_{mod} \quad \text{in RF } S,$$

where ν_0 is the signal frequency emitted by a fixed in S source.

Therefore for the observed blue shift we have

$$\Delta\nu_{obs} = \nu' - \nu_{mod} = \nu_0 \frac{wt}{c}, \qquad (8),$$

and ν_{mod} is the frequency defined by neglect of non-inertiality of S'. In the approximation considered the shift is linear in time. The rate of shift $\dot{\nu}_{obs} = \frac{d\nu_{obs}}{dt}$ is defined by the following simple relation

$$\dot{\nu}_{obs} = \nu_0 \frac{w}{c}. \qquad (9)$$

This result, in principle, is well known. Here we have to do, in fact, with the effect of the gravitational (Einstein) frequency drift described on the basis of the equivalence principle. The shift is blue because of the non-inertial observer and the source of signal approach each other and therefore the magnitude of the equivalent gravitational potential at the observation point stands out about its magnitude at the point of the signal emission.

The acceleration x-component $W'_x = \frac{d^2x'}{dt'^2}$ transformation law can be derived directly from (6). Elementary calculus in the comoving RF ($V_x = 0$) gives

$$W'_x = \frac{(\xi^2 - \eta^2)^4}{(\xi^2 + \eta^2)^3}(W_x - \xi w).$$

In the approximation of $\xi \approx 1,\ \eta \ll 1$ we have therefrom

$$W'_x = W_x - w. \qquad (10)$$

This relation holds in every comoving RF as long as in this frame $\xi \approx 1,\ \eta \ll 1$ approximation is valid. A probe particle free of dynamical influence in the comoving RF, i.e. with $W_x = 0$, in accordance with (16) will be uniformly accelerated in the non-inertial RF S' with the constant acceleration of $-w$. Such an acceleration can be registered by any observer fixed in any point of this non-inertial RF S'. By the equivalence principle the non-inertial observer is entitled to identify this acceleration with an existence of a constant (background) gravitational field which results in acceleration w.

3. Conformal deformation of the light cone and time inhomogeneity

Now we consider the transformations of the light cone generatrices under SCT (1). Because of $x'^\mu x'_\mu = x^\mu x_\mu (1+2(ax)+(a)^2(x)^2)^{-1}$ transformations (1) leaves the light cone equation invariant, i.e. from $x^\mu x_\mu = 0$ follows $x'^\mu x'_\mu = 0$. However the light cone surface is deformed non-linearly. From (1) generally

$$x'^\mu = \frac{x^\mu}{1 + 2(ax)}. \qquad (11)$$

when additionally $x^\mu x_\mu = x'^\mu x'_\mu = 0$.

In the two-dimensional case considered we have the relation

$$t'_\pm = \frac{t}{1 \pm \frac{t}{t_{lim}}}, \qquad (12)$$

where $t_{lim} = c/w$.

The sign choice corresponds to signal propagation in the forward and backward directions. We are reminded that the symbol t (correspondingly t') represents time of a light signal propagation between two spatially separated points in the space of Lorentz RF S (correspondingly in the non-inertial RF S'). So the quantities $R = ct$ and $R' = ct'$ define location distances in both of these RFs.

Obviously a semi-infinite time interval $0 \leqslant t < \infty$ corresponding to the positive (forward) direction of signal propagation maps onto a finite time interval $0 \leqslant t'_+ \leqslant t_{lim}$. For the backward direction on the contrary a finite interval $0 \leqslant t \leqslant t_{lim}$ maps onto a semi-infinite time interval $0 \leqslant t'_- < \infty$ (Fig. 2).

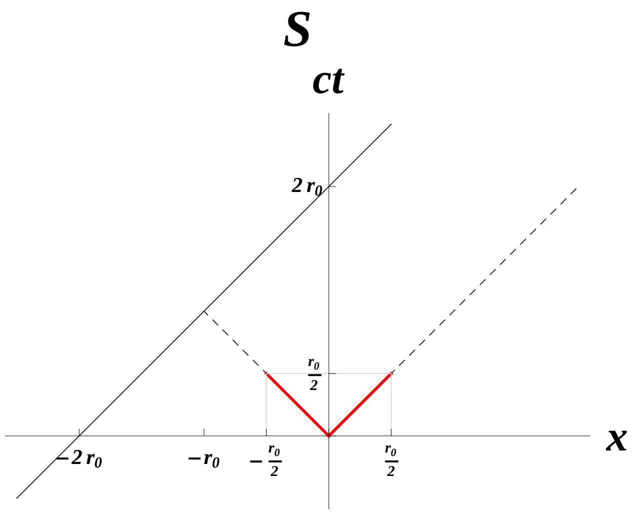

Figure 2. Conformal deformation of the light cone. Initially equal intervals $t_{\pm} = \frac{t_{lim}}{2}$ of forward and backward generatrices transforms into unequal intervals $t'_+ = \frac{t_{lim}}{3}$ and $t'_- = t_{lim}$.

The non-linear time transformation (12) further will be referred to as the conformal deformation of time or conformal time inhomogeneity.

4. The dependence of distance on red shift. The Hubble law

First and foremost we show that transformations (12) allows us to receive an explicit expression for location distance as a simple function of red shift z.

Let us consider on the basis of the formula (12) the case of a signal propagation from the deep past to the point of the observer position. That mean that we are to choose the lower sign in the formula (12):

$$t' = \frac{t}{1 - \frac{t}{t_{lim}}}, \tag{13}$$

where $t_{lim} = c/w$ and t is the time of the signal propagation to the point of observation.

First of all we are to receive from (13) the explicit expression for the time interval of the signal propagation in the form of a function on the red shift z.

We have from (13) for small time increments $\Delta t'$ and Δt the following expression

$$\Delta t' = \Delta t(1 - \frac{t}{t_{lim}})^{-2}. \tag{14}$$

If Δt and $\Delta t'$ are the periods of oscillations of emitted ($\Delta t = T_{emitted}$) and received ($\Delta t' = T_{observable}$) signals correspondingly, then using the standard definition of red shift

$$\frac{\lambda_{observable}}{\lambda_{emitted}} = z + 1, \tag{15}$$

where $\lambda_{observable} = cT_{observable}$ and $\lambda_{emitted} = cT_{emitted}$, we find from (14) the expression

$$\frac{\lambda_{observable}}{\lambda_{emitted}} = (1 - \frac{t}{t_{max}})^{-2} = z + 1, \tag{16}$$

which gives

$$t(z) = t_{max}\frac{(z+1)^{1/2} - 1}{(z+1)^{1/2}}. \tag{17}$$

Here $t(z)$ represents the time interval between the moments of emitting and receiving the light (electromagnetic) signal. So, supposing that the speed of light is constant and does not depend on the velocity of the emitter, the quantity $R = ct$ can be regarded as the distance covered by the signal.

From the formula (17) we obtain an expression which determines the explicit form of dependence of R on the red shift z[1]:

$$R(z) = R_u\frac{(z+1)^{1/2} - 1}{(z+1)^{1/2}} = R_u\left(1 - \frac{1}{(z+1)^{1/2}}\right). \tag{18}$$

Here$R_u = ct_{lim}$ is a parameter, which, within in the model suggested, has the sense of the limit (maximal) distance.

The quantity $R(z)$ defined by (18) corresponds to the distance, which in cosmology is referred to as a location distance. In principle the relation (18) allows direct experimental verification in the whole range of z variation and can be confirmed or refused by observations.

Now we can obtain explicit expression for the Hubble law. For this purpose we make use of the well-known formulae describing the Doppler effect in Special Relativity. The explicit expression for the longitudinal Doppler effect immediately follows from the Lorentz boosts with due regard for thew light-cone equations.

The well known expression for the relative velocity of the mutually recessing emitter and receiver of the signal $V(z)$ is as follows:

$$\frac{V(z)}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}. \tag{19}$$

This expression is valid in the whole range of velocity V variation.

We again emphasize the essentially kinematic nature of the relation (18). It is the manifestation of the nonlinear conformal time deformation (12) which follows from Special Conformal Transformations exactly in the same manner, as the Doppler effect and the dependence $V(z)$ follows from the linear time deformation, which arise from the Lorentz boosts leaving the equation of light cone unaltered.

Now we can find using (18) and (19) the following expression for the ratio V/R:

$$\frac{V(z)}{R(z)} = cR_u^{-1}f(z), \tag{20}$$

$$f(z) = \frac{(z+1)^{1/2}}{(z+1)^2 + 1}\cdot\frac{(z+1)^2 - 1}{(z+1)^{1/2} - 1}. \tag{21}$$

It is easy to see that $\lim_{z\to 0} f(z) = 2$.

In this limit we receive the conventional expression for the Hubble law:

$$V = H_0 R, \tag{22}$$

[1] Relation of the type $R(z) = const \cdot z$, that connect cosmological distance R with the red shift, seems to be firstly obtained from the conformal symmetry arguments by Ingraham [8] in 1954.

where H_0 is the Hubble constant.

By comparing this formula with the expression for the obtained $\lim_{z\to 0}\frac{V(z)}{R(z)}$ from formula (20) we can establish the following connection between the acceleration w and the Hubble constant H_0:

$$2cR_u^{-1} = 2w = cH_0. \tag{23}$$

It is seen that the relation (12) defining the conformal deformation of time (the time inhomogeneity) allows us to establish the following simple connection between the parameter w defining the background acceleration and the Hubble constant H_0

$$w = \frac{1}{2}cH_0. \tag{24}$$

Hence in the approach considered the constant acceleration w intrinsic to non-inertial RF S' can be naturally connected to the Hubble constant H_0, defining space expansion.

5. Time inhomogeneity and the blue-shifted frequency drift. The Pioneer anomaly

Now let us consider on the basis of the formula (12) the location-type experiments. The conventional scheme of such an experiment is as follows:

(1) the signal is emitted from the point of the observer location at the time instant t_A^0,

(2) the signal is arrived and reemitted at the time instant t_B,

(3) the signal is returned to the observer at the time instant t_A.

Under the assumption of the coincidence of the forward $(t_B - t_A^0)$ and backward $(t_A - t_B)$ time intervals one can receive the formula for the signal traveling time $t = \frac{1}{2}(t_A - t_A^0)$ and then accept the formula $R = ct$ for the corresponding location distance.

The time inhomogeneity (12) changes situation so that the forward and backward time intervals do not coincide. The last time interval is larger then the first one (Fig. 3).

In application to real experiment analysis one should use (12) for the small time intervals i.e. when $\Delta t/t_{lim} = (t_A - t_A^0)/t_{lim} \ll 1$. In this case formula (12) gives to the second order of t/t_{lim}

$$t'_{\pm} = t \mp \frac{t^2}{t_{lim}}. \tag{25}$$

In accordance with the location distance definition the length of the signal travel in forward and backward directions will be

$$x'_{\pm} = ct'_{\pm} = x \mp \Delta x = x \mp \frac{W_0}{2}t^2, \tag{26}$$

where

$$W_0 = \frac{2c}{t_{lim}} = 2w. \tag{27}$$

Therefore in the $t/t_{lim} \ll 1$ approximation (t is the time of signal propagation) the forward and backward location distances $x'_{\pm}$ differ from $R = ct$ by $\Delta x = \frac{W_0}{2}t^2$. From the usual point of view it seems like the emitter fixed in any space point suffers constant acceleration $W_0 = 2w$ directed to the observer.

Clearly $t/t_{lim} \ll 1$ condition is equivalent to the condition $\frac{wt}{2c} \ll 1$, $\frac{wx}{2c^2} \ll 1$ (see Sec. 1), which formally corresponds to Galilei-Newton kinematics. So the predicted effect in the location-type

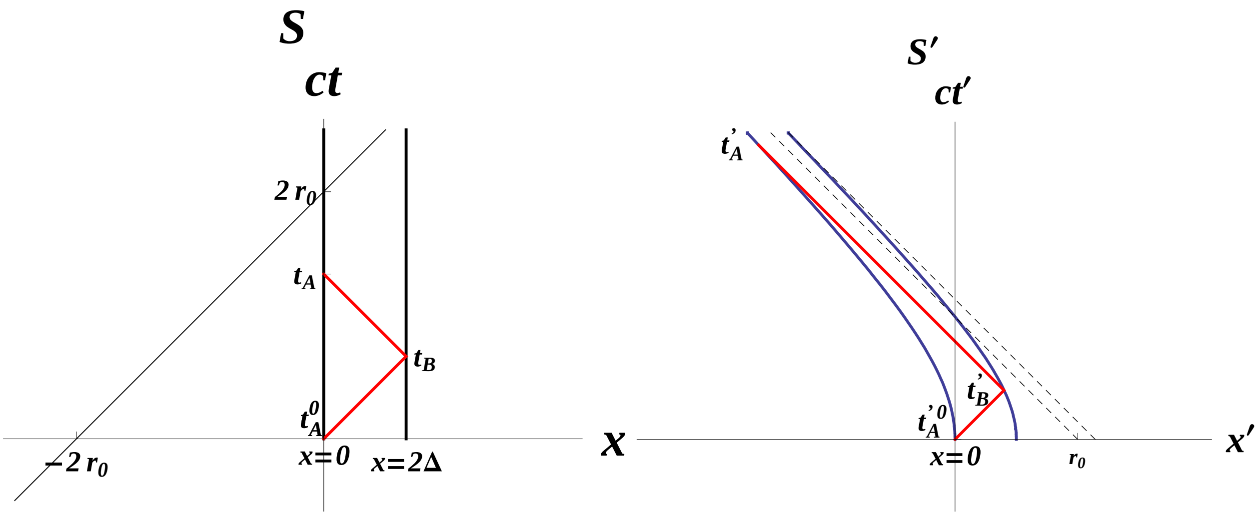

Figure 3. Location-type experiment. World lines and dashed lines are as on Fig. 1. Thin lines is a light signal world lines. Because of conformal transformations light lines remains straight and inclined at 45°. In RF S we have $t_A - t_B = t_B - t_A^0$ but in S' always the inequality $t'_A - t'_B > t'_B - t'^0_A$ takes place.

experiments will be the blue frequency shift, value of which will be defined by the formula similar to (8), i.e.

$$\Delta\nu_{obs} = \nu_0 \frac{W_0 t}{c}. \tag{28}$$

Due to (27) value of this shift is twice predicted by (8).

For the constant rate of frequency shift $\dot{\nu}_{obs} = \frac{d(\Delta\nu_{obs})}{dt}$ we find the following relation analogous to (9):

$$\dot{\nu}_{obs} = \frac{1}{c}\nu_0 W_0. \tag{29}$$

As by (27) $W_0 = cH_0$, where H_0 is the Hubble constant, then from (29) we have

$$\dot{\nu}_{obs} = \nu_0 H_0. \tag{30}$$

This relation defines frequency drift as a function of a fixed emitter frequency ν_0. This result can be confirmed via generalization of the standard SR clock synchronization procedure to the space expanding case [9,10].

According to the approach under consideration the anomalous blue-shifted drift is the consequence of the background acceleration existence (i.e. non-inertiality of the observers RF). It can be observed in principle under suitable conditions (in the absence of any gravitating sources) on any frequency even in the case of mutually fixed emitter and receiver. From this point of view PA should be treated as the first clearly observed effect of that kind. The experimental data of Pioneer tracking can be used for determination (or at least for estimation) of the parameter t_{lim} and corresponding background acceleration.

On the other hand the uniform blue-shifted drift $(\dot{\nu}_{obs})_P$ is measured experimentally with a great accuracy $(\dot{\nu}_{obs})_P = (5.99 \pm 0.01) \cdot 10^{-9}$ Hz/s [1–4]. Therefore it can form a basis for the new (alternative to the cosmological observations) high precision experimental estimation of the numerical value of the Hubble constant.

For that goal we make use of (30) and recall that frequency of Pioneer tracking is $(\nu_0)_P = 2.29 \cdot 10^9$ Hz, so

$$H_0 = \frac{(\dot{\nu}_{obs})_P}{(\nu_{obs})_P} \cong 2.62 \cdot 10^{-18}\ \mathrm{s}^{-1}, \tag{31}$$

what is consistent with generally accepted value of $H_0 \cong 2.4{\cdot}10^{-18}$ s^{-1} obtained from cosmological observations.

For the "acceleration" a_P we have

$$a_P = cH_0 = 7.85 \cdot 10^{-8} \text{ cm/s}^2,$$

what is in the range of uncertainty of PA data ($a_P^{exp} = (8.74 \pm 1.33) \cdot 10^{-8} \frac{\text{cm}}{\text{s}^2}$).

The numerical coincidence of the results can be considered as experimental evidence of anomalous blue-shifted drift as kinematical manifestation of the conformal time inhomogeneity. In other words from the point of the considered approach the measured in experiments of electromagnetic wave propagation quantities favor relation (28) (but not (8)) for the anomalous frequency shift.

It should be stressed that physical meaning of the relations (8) and (28) in spite of their visual similarity is fundamentally different.

Equation (8), defining blue frequency shift by the non-inertiality of RF S', in fact was obtained in Galilei-Newton kinematics. There time transformation under transition from RF S to S' has the form $t = t'$ (see (4)), and the velocity transformations (7) and the acceleration w are defined as usual in Galilei-Newton kinematics.

On the other side, formula (28) was obtained from the exact non-linear time transformation (12) defining the time inhomogeneity, so outside of Galilei-Newton kinematics. "Constant acceleration" $W_0 = 2w$ appears due to the quadratic character of the first non-linear term in the series expansion of $t'(t)$ in terms of small parameter t/t_{lim} by (12), while location distance is defined as $R' = ct'$.

Hence "acceleration" W_0 is not a "truly" acceleration (i.e. its origin is not a force or a dynamical source) but rather "mimic" acceleration. This imitation appears because of the effects originated by non-linearity of time course (time inhomogeneity) is interpreted in terms of traditional paradigm of time homogeneity.

The possibility of assigning the anomalous frequency drift observed in the signals transmitted by Pioneer 10/11 to the quadratic time inhomogeneity was noted in the first comprehensive works on PA [1] and [2]. However there were no mention for theoretical reasons for such a phenomenological approach.

Concerning the acceleration parameter $w = \frac{1}{2}cH_0$ we note that it seems to be a strictly natural candidate for the "minimal acceleration" of Milgrom, which is a fundamental dynamical parameter of the modified Newton Dynamics (MOND) (see [11,12]), which in turn is a phenomenological alternative for the Cold Dark Matter approach. This question will be subject of a separate publication.

6. The possible kinematic origin of the accelerating Universe phenomenon

Now we analyze the general relation (20) for $V(z)/R(z)$

$$\frac{V(z)}{R(z)} = cR_u^{-1} \frac{(z+1)^{1/2}}{(z+1)^2+1} \cdot \frac{(z+1)^2-1}{(z+1)^{1/2}-1}.$$

Taking into account the connection (23) between R_u and the Hubble constant H_0, we rewrite this equation in the dimensionless form as

$$\phi(z) = \frac{V(z)}{H_0 R(z)} = \frac{1}{2} \frac{(z+1)^{1/2}}{(z+1)^2+1} \cdot \frac{(z+1)^2-1}{(z+1)^{1/2}-1}. \quad (32)$$

The function $\phi(z)$ and its derivative $\phi'(z) = \frac{d\phi(z)}{dz}$ are shown in Fig. 4 and Fig. 5. Horizontal line in Fig. 5 represents strict Hubble law (34). We see that this function possesses a maximum

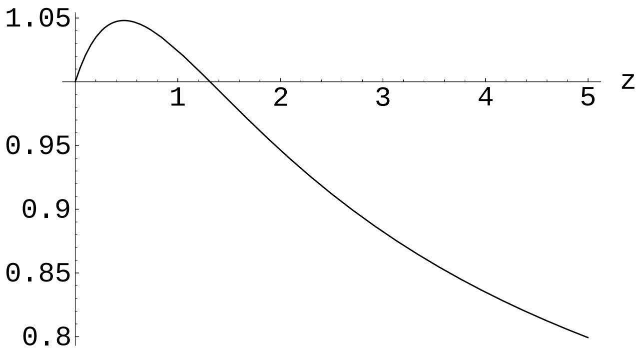

Figure 4: The function $\phi(z)$.

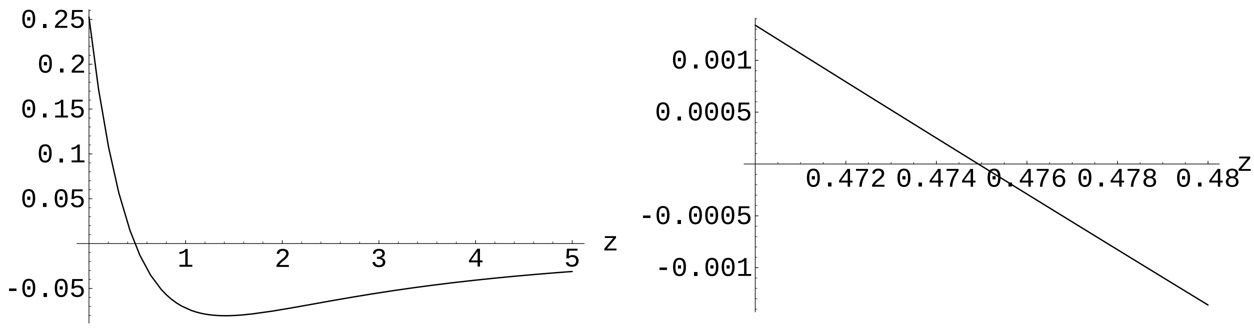

Figure 5: The derivative $\phi'(z)$ and position of its zero.

at $z_0 \cong 0.475$ (see Fig. 4 and Fig. 5). Overall variation of $\phi(z)$ demonstrates that in the interval $0 \leq z < z_0$ the distance $R(z)$ increases with z more slowly, and in the interval $z_0 < z < \infty$ approaches its limit value R_u more rapidly, than the velocity $V(z)$ approaches its limit c.

As regards a possible treatment of the behavior of the function $\phi(z)$ in terms of the standard dynamical GR approach using the decelaration parameter, we are to notice the following. According to the pure kinematic approach proposed in our paper, the source of the effects induced by the cosmologic expansion is the conformal time inhomogeneity. The "acceleration" attributed to the emitting source arises because of treating the actual nonlinear time dependence $t'(t)$ in terms of the traditional theoretical paradigm based on the time homogeneity concept. The uniform "acceleration" $W_0 = cH_0$ which appears in the formula (26) in the $t/t_{lim} \ll 1$ approximation is not a "true" but the "effective" acceleration in the reality. Such an "acceleration" in the general case must be treated as time-dependent one. Pure formally it can be determined as the second derivative on time of the function

$$R(t) = ct' = ct\left(1 - \frac{t}{t_{lim}}\right)^{-1} \tag{33}$$

and it possess the following form

$$W(t) = \frac{2c}{t_{lim}}\left(1 - \frac{t}{t_{lim}}\right)^{-3}. \tag{34}$$

Such a "time-dependent effective acceleration" is directed to the point of observation and coincides in the first approximation in t/t_{lim} with $W_0 = cH_0$. This "acceleration" can be presented according to (16) in the form

$$W(z) = W_0\,(1+z)^{3/2}.$$

Thus one can say that the numerical value of such an "acceleration" decreases during the process of the Universe expansion starting from the very large magnitude ($z \gg 1$).

Evidently, the interpretation of $\phi(z)$ behavior from the point of view of common treatment seems as follows. In the interval $\infty > z > z_0$ there is the deceleration $\frac{d\phi}{dz} < 0$ of cosmological expansion, and at the point of $z_0 \cong 0{,}475$ it changes to the acceleration $\frac{d\phi}{dz} < 0$. Numerical value of z_0 agrees quite well with experimentally founded "point of change" $z_{exp} = 0.46 \pm 0.13$ from the deceleration of cosmological expansion to the acceleration.

7. Concluding remarks

It should be emphasized that the basic formula (12) for the conformal transformations of the time, as well as all its consequences are valid on the assumption that the Hubble parameter H_0 is constant. Hopefully this assumption is reasonable as applied to at least later stages of the Universe evolution. In this case the proposed formulae (18) and (20) can be valid for the experimentally obtained values of the read shift having the order of several units.

Needless to say that the assumption that the Hubble constant is time-independent does not mean that the observable magnitude of the Hubble constant is its lower limit. Any rigorous theoretical arguments in favour of such an assumption are absent today. Only some reasoning which are noting more than a suggestive ones may be cited. They are connected with so-called Mass Dependent Maximal Acceleration (MDMA) concept (see [13] and references within) together with the Maximal Tension (or Maximal Force) hypothesis firstly proposed in [14] and independently in [13]. If we define, following Gibbons [14], the Maximal Force F_0 as $F_0 = \frac{c^4}{4G}$ (G being the Newton gravitational constant) the corresponding MDMA for the mass M will be defined as $W(M) = \frac{F_0}{M}$. Putting the Universe "diameter" equal to $R_u = 2cH_0^{-1}$ and defining the Universe "mass" M_u as a product of the critical density $\rho_c = \frac{3H_0^2}{8\pi G}$ and the Universe "volume" $V_u = \frac{4\pi}{3}\left(\frac{R_u}{2}\right)^3$ we obtain $W(M_u) = \frac{1}{2}cH_0$ i.e. the value coinciding with the background acceleration magnitude.

It is self-evident that such an intriguing coincidence is to be confirmed in the frame of some consistent theoretical scheme. The problem connected with the maximum tension hypothesis as applied to the cosmological problems call for special considerations in further publications of the author.

The author would like to acknowledge E.V. Doktorov, V.V. Kudryashov, A.E. Shalyt-Margolin, Ya.M. Shnir, I.A. Siutsou and Yu.P. Vybly for their invaluable critical remarks and for the fruitful discussions.

The paper is in part supported by the Belarus Foundation for Fundamental Research (BFFR), Project F06-129.

References

[1] Anderson J.D. et al. Phys. Rev. Lett. **81**. 2858–2861 (1998); gr-qc/9808081.
[2] Anderson J.D. et al. Phys. Rev. D. **65**. 082004/1–50 (2002); gr-qc/0104064.
[3] Turyshev S.G., Nieto M.M., Anderson J.D. gr-qc/0503021.
[4] Turyshev S.G., Nieto M.M., Anderson J.D. gr-qc/0510081.
[5] Reiss A.G., et al. Astrophys. J., **607**. 665–687 (2004).
[6] Perlmutter S., et al. Astrophys. J., **517**. 565–586 (1999).
[7] Fulton T., Rohrlich F., Witten L. Nuovo Cim. **34, N 3**. 652–670 (1962).

[8] Ingraham R.L. Nuovo Cimento. **12, N 6**. 825–850 (1954).
[9] Tomilchik L.M. gr-qc/0704.2745.
[10] Tomilchik L.M. Optics and Spectroscopy. **103, N 2**. 237–241 (2007).
[11] Milgrom M. Astrophys. J. **270**, 365 (1983).
[12] Bekenstein J. and Milgrom M. Astrophys. J. **286**, 7 (1984).
[13] Tomilchik L.M., Kudryashov V.V. Proc. Int. School-Sem. "Actual Problems of Microworld Physics" (Gomel, Belarus, July 28 – Aug. 8, 2003). V1. 24. JINR, Dubna (2004); gr-qc/0507090.
[14] Gibbons G.W. Found. Phys. **32**. 1891 (2002); hep-th/0210109.