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Muon multiplicity at high energy proton-nuclei collisions

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Estimation of multiplicity of muons and pions production at high energy proton-nuclei collisions is given. Both QED and QCD contributions are considered for peripheral kinematics of muon pair and σ-meson production, keeping in mind it's final conversion to muons. An attempt to explain the excess of positive charged muons compared to negative one in cosmic muon showers is given. We derive the dependence of cross-section of n pairs as a function of n at large n as $d^n(n!n^2)^{-1}$.

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1. Introduction

The essential impact parameter in pair of charged particle (muons or pions) creation process at collision of high energy particle (proton or nuclei from cosmic ray) is of the order $\frac{1}{m_\pi} \sim \frac{1}{m_\mu} = \frac{1}{m} \sim 2fm$. Such a particle penetrating through the Earth atmosphere at least once will collide the nuclei of gas (nitrogen or oxygen). It corresponds to direct collision with small orbital momentum or impact factor $\rho < 1fm$. Besides the pair can be created in peripheral collisions which are responsible for large values of orbital momentum $\rho > 1fm$. It is interesting to evaluate the number of high energy muons which reach the surface of the Earth. In particular the problem of explaining the exceed of positive muons or rather high energies over the negative ones discussed first in [1]. This number is the sum of the numbers of created muons and pions, as well all the pions due to main decay mode will turn to muons. As for creation of light lepton pairs (electron and positron) their fate is to create the showers which do not produce muons. The probability of creation of heavy lepton pairs such as tau-meson is suppressed compared with muons one by factor $(m/m_\tau)^2 \sim 10^{-3}$ and will not be considered below.

2. QED mechanism of muons production

Cross section of production $\mu^+\mu^-$ pair at high energy proton-nuclei collision have a form

$$\sigma^{QED}_{(1)} = \frac{28(Z\alpha^2)^2}{27\pi m^2} \times$$
$$\times[L^3 - 2.2L^2 + 3.8L - 1.6 - 3L^2 f(Z\alpha)], \qquad (1)$$
$$f(t) = t^2 \sum\nolimits_1^\infty \frac{1}{n(n^2+t^2)} \qquad (2)$$

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with $L = \ln(2\gamma), \gamma = \frac{E}{M}, E, M$- energy and mass of proton in laboratory frame, Z is the nuclei charge and m is the muon mass. The first 4 term in square bracket (1) was obtained by Racah [2], the last one, which takes in to account many photon exchanges was obtained by Bethe and Maximon [3].

For $L = 15$, $Z = 7$ we have $\sigma_{(1)}^{QED} \approx 5 \times 10^{-30} cm^2$. We estimate the number of muon pairs created by a single proton as $N_\mu^{QED} \approx M_P^2 \sigma_{(1)}^{QED} \approx 10^{-2}$.

Production of several pairs of charged particles contribution to the cross section is not associated with L^3 enhancement. It was shown in [5] that the distribution on the impact parameter of probability of n lepton pairs production in ion-ion collisions have a Poisson form:

$$\frac{d\sigma_n}{d^2\rho} = P_n(A_l), \qquad P_n(z) = \frac{z^n}{n!} e^{-z}, \tag{3}$$

with

$$A_l = \frac{28(Z\alpha^2)^2}{9\pi^2\rho^2 m^2} \Phi(\rho, \gamma), \tag{4}$$

ρ-impact parameter-the distance in transversal direction between proton and nuclei, $\Phi(\rho, \gamma)$ will be given below.

This result can be generalized for the case of production of arbitrary number of lepton n_l and pion n_π pairs with fixed total number $n = n_l + n_\pi$:

$$\frac{d\sigma_n^{QED}}{d^2\rho} = \frac{1}{n!}(A_l + A_\pi)^n e^{-A_l - A_\pi}, n = 2, 3, ... \tag{5}$$

with [4] $A_\pi \approx A_l/14$. This estimation is based on the ratio of leading cross sections in Born approximation and the relation $1/m_\pi^2 \approx 1/(2m^2)$. This results in replacement $A_l + A_\pi$ by A_l and the coefficient 28/9 in expression for A_l by 10/3.

The quantity $\Phi(\rho, \gamma)$ have a form [6]:

$$\begin{aligned} \Phi(x, \gamma) &= \ln \frac{\gamma^2}{x} \ln \frac{x}{x_0} + \frac{1}{4} \ln^2 \frac{x}{x_0}, \quad x_0 < x < \gamma^2, \\ \Phi(x, \gamma) &= (\ln \gamma^2 - \frac{1}{2}(\ln x_0 + \ln x))^2, \quad \gamma^2 < x < \frac{\gamma^4}{x_0 x}, \\ x &= (m\rho)^2, \quad x_0 = (mR)^2, \end{aligned} \tag{6}$$

with R-nuclei radii.

The cross section of creation of n pairs in the single collision will be

$$\sigma_n^{QED}(\gamma) = \frac{\pi B^n}{m^2 n!} I_n(\gamma, B), \quad n > 1, \tag{7}$$

with $B = \frac{10}{3\pi^2} L (Z\alpha^2)^2$ and

$$I_n = \int_{x_0}^{\infty} \frac{dx}{x^n} \Phi^n e^{-\frac{B}{x}\Phi}. \tag{8}$$

For $L = 15, Z = 7$ we have $B \approx 7.05 \times 10^{-7}$. The Quantity $I_n(\gamma, A)$ for $x_0 = (R(fm))^2/4 \approx 1$ can be approximated as $I_2 \approx 8L^2; I_3 \approx 3L^3; I_4 \approx 1,58L^4, ...$

For the case $n = 1$ we must take into account the unitary corrections to the cross section in Born approximation which will be considered below.

$$\sigma_1^{QED} = \sigma_B^{QED} + \frac{B}{m^2} \int \frac{d^2\rho}{\rho^2} \Phi(e^{\frac{-B\Phi}{\rho^2}} - 1) \mid_{\rho > R} . \tag{9}$$

3. Pomeron exchange mechanism of muons production

In a simplified Pomeron model [7] the exchange by two reggeized gluons in the scattering channel is associated with Pomeron Regge pole exchange. Pomeron-proton coupling is described by the effective vertex

$$\Phi_P(\vec{l}_1,\vec{l}_2)^{\lambda_1\lambda_2} = -\frac{12\pi^2}{N_c}F_P(\vec{l}_1,\vec{l}_2), \quad N_c=3;$$
$$F_p(\vec{l}_1,\vec{l}_2) = \frac{(-3\vec{l}_1\vec{l}_2)C^2}{[C^2+(\vec{l}_1+\vec{l}_2)^2][C^2+\vec{l}_1^2+\vec{l}_2^2-\vec{l}_1\vec{l}_2]},$$
$$C = m_\rho/2; \tag{10}$$

$\vec{l}_1, \vec{l}_2 = \vec{l}-\vec{l}_1$-are two-dimensional vectors of gluons.

The vertex of emission of σ-meson in collision of two Pomerons can be obtained using the $RRPP$ effective vertex obtained in the paper [8]. Really one can consider only the kinematic region when the 4-momenta of the gluons almost equal $p_1 = p_2 \approx (q_1+q_2)/2 = p/2, \quad p^2 = M_\sigma^2$ with $q_{1,2}$-the momenta of reggeized gluons, M_σ-is the σ-meson mass. Projecting this vertex on the colorless and spin-zero state we obtain:

$$\frac{1}{\sqrt{N_c^2-1}}\delta^{a_1a_2}g^{\nu_1\nu_2}\Gamma^{-\nu_1\nu_2+}_{ca_1a_2d}(q_1;p_1,p_2;q_2) =$$
$$\frac{-4\pi\alpha_s}{\sqrt{N_c^2-1}}N_c\delta^{cd}I(q_1,q_2), \tag{11}$$

with

$$I(\vec{q}_1,\vec{q}_2) = 12 - \frac{8}{M_\sigma^2+2\vec{q}_1^{\,2}+2\vec{q}_2^{\,2}}\times$$
$$\times\left[10(M_\sigma^2+\vec{q}_1^{\,2}+\vec{q}_2^{\,2}) - \frac{5}{2}(M_\sigma^2+(\vec{q}_1+\vec{q}_2)^2)+\right.$$
$$\left.\frac{16\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{M_\sigma^2+(\vec{q}_1+\vec{q}_2)^2}\right]. \tag{12}$$

Matrix element of process of single σ-meson production in proton-proton collisions have a form

$$M^{pp\to pp\sigma}(l,p) = isA_1 f(l,p)F(\Delta)\frac{2^7\alpha_s^3\pi^2N_c}{\sqrt{N_c^2-1}}, \tag{13}$$

with $A_1 = A - A^{1/3}$ is the number of nucleons inside the nuclei with atomic number A which interact with the high energy proton by Pomeron exchange,

$$f(l,p) = \int\frac{d^2l_1C^4}{2\pi\vec{l}_1^{\,2}(\vec{l}-\vec{l}_1)^2(\vec{p}-\vec{l}_1)^2}F_P(l_1,l-l_1)\times$$
$$\times F_P(l_1-l,p-l_1)I(\vec{l}_1,\vec{p}-\vec{l}_1), \tag{14}$$

and form-factor of the two gluon bound state $F(\Delta) = [a^2\Delta^2+1]^{-2}$, the relative momentum of gluons $\Delta = |\vec{p}_1-\vec{p}_2|/2$ and a is the size of two gluons bound state $a \approx 1fm$. Performing the phase volume of the final 4-particle state as

$$d\Gamma_4 = \frac{d^3p'_a}{2E_a}\frac{d^3p'_b}{2E_b}\frac{d^3p_1}{2\omega_1}\frac{d^3p_2}{2\omega_2}(2\pi)^{-8}\times$$
$$\times\delta^4(P_a+P_b-P'_a-P'_b-p_1-p_2)$$
$$= \frac{d^2l_1d^2p}{(2\pi)^22s}L\frac{d^3\Delta}{(2\pi)^3M_\sigma}(2\pi)^{-3} \tag{15}$$

After integration over Δ

$$\int \frac{d^3\Delta}{(2\pi)^3 M_\sigma} F^2(\Delta) = \frac{1}{32\pi M_\sigma a^3}$$
$$= \frac{M_p^3}{32\pi 5^3 (a(fm))^3 M_\sigma}. \quad (16)$$

The cross section of sigma meson production in Born approximation can be written in form:

$$\sigma_{01}^p = \frac{9A_1^2\alpha_s^6 L}{4000}\frac{M_p^3}{M_\sigma m_\rho^4} J, \quad (17)$$

$$J = \int \frac{d^2l d^2p}{(2\pi)^2 C^4} f^2(l,p) \approx 7.4 * 10^3. \quad (18)$$

Numerical estimation gives for $M_\sigma \approx C \approx 400 MeV$,
$J \approx 1$.

Screening effect are taken into account below(see (20), n=1). Consider now process of several σ-meson production at proton-nuclei peripheral collisions.

At large impact parameters limit proton interact with the whole gluon field of the nuclei coherently. So main contribution arises from many Pomeron exchanges mechanism (compare with the "chain" mechanism essential at BFKL equation formation). The relevant Feynman diagrams-are the s-channel iteration of Pomeron exchange. Let us consider three kinds of iteration blocks. One is the pure Pomeron exchanges, the second one-Pomeron exchange with the vertex of emission of external σ- meson insertion. The third one-so called "screening block"-two blocks of second type with common virtual σ-meson Green function. Contribution of the last one is associated with boost logarithm-in quite analogy with the problem of several leptonic pairs production at ions collision considered in paper of one of us [5]. In the similar way the closed expression for the summed on numbers of ladders of the first and the third type can be obtained using the relation

$$\int \Pi_1^n \frac{d^2k_i}{(2\pi)^2} = \int \Pi_1^{n+1}\frac{d^2k_i}{(2\pi)^2}\int d^2\rho e^{i\vec\rho \sum_1^{n+1}(\vec k_i - \vec q)}$$
$$= \int d^2\rho e^{-i\vec q\vec\rho}\Pi_1^{n+1}\frac{d^2k_i}{(2\pi)^2} e^{i\vec k_i\vec\rho}. \quad (19)$$

Omitting the details which are similar to ones given in [5] we arrive to the expression for the cross section

$$\sigma_n^P = \frac{A_1^2}{m_\rho^2}\int \frac{C^2 d^2\rho}{2\pi}\frac{z^n}{n!}e^{-z}, \quad n = 1,2,3,... \quad (20)$$

with

$$z(\rho) = \frac{9L\alpha_s^6 M_p^3 D}{4*10^3(a(fm))^3 M_\sigma m_\rho^2},$$
$$D = \int \frac{d^2l d^2p}{(2\pi)^2 C^4} f^2(l,p) e^{i\vec p\vec\rho} \approx e^{-C\rho} J. \quad (21)$$

Let us estimate the behavior of σ_n^P at large n. Keeping in mind the numerical smallness of z in (20) we have

$$\frac{\sigma_n^P}{\sigma_1^P} \approx \frac{d^n}{n^2 n!}, d \approx 0.000425L, a \approx 1fm. \quad (22)$$

4. Excess of positively charged muons produced in cosmic rate interaction with the Earth

Let us apply the results obtained above to the problem of explain of exceeding number of positive charged muon compared the negative charged muons created by cosmic ray (preferentially consisting from protons).

Rather rough estimation leads to conclusion that a high energy proton crossing the atmosphere normally to Earth surface averagely in 1.2 events collide with nuclei of a gas (nitrogen or oxygen). It excite the nucleons which are on it's way through the nuclei and interact peripherally with the other nucleons. Number of the first ones is $N_d \approx A^{1/3}$, number of others is $N_p \approx A_1 = A - N_d$. Resonances decay product is mainly positive charged pions and the last ones reach the Earth surface as positive charged muons. We believe that the direct type collisions corresponds to values of impact parameter of order of $1fm$-the proton radius. For peripheral collisions we choice $\rho > 1fm$.

The peripheral interactions produce equal number of positive and negative pions (muons). The number of the last ones is (we choose $Z = 7, A = 14, L = 15$):

$$N_p = M_p^2(\sigma_1^{QED} + \sigma_1^P) \approx M_p^2 \sigma_1^P \approx 7.8, \tag{23}$$

where we can consider the only cross section of one pair or one σ meson with subsequent decay to the pair of charged pions.

Really the relative contribution of two pairs with impact parameter exceeding $\rho > 1fm$ is suppressed by additional factor $exp(-\rho m_\rho/2)|_{\rho\sim 1fm}$.

Contribution of distant objects are suppressed by factor $exp(-Rm_\rho/2) < 10^{-5}$ for $R > 5fm$. So the importance of distant spectators become negligible.

It is known [9] that the ratio of positive charged muons to the number of negative charged muons created in collision of the high energy cosmic rays in the "knee" region of the spectrum $\Gamma = 10^6$ with nuclei of the atmospheric gas exceed unity:

$$R_{exp} = \frac{N_{\mu^+}}{N_{\mu^-}} = 1.374 \pm 0.004(stat.)^{+0.012}_{-0.010} \tag{24}$$

The result obtained within our approach, $R_{th} = 1 + (N_d/N_p) \approx 1.32$, is in a reasonable agreement with R_{exp}.

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