Nonlinear Dynamics and Applications. Vol. 14 (2007) 20 - 24

The Monte-Carlo Simulation of Heavy-Ion Collisions

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We study the multiplicity distributions of charged particles in high energy heavy-ion collision with the help of the Ginzburg-Landau model for first-order QGP→ hadron phase transition and the Monte-Carlo simulation. Parameters of the model and the values of scaling exponent are found.

PACS numbers: 12.38.Mh, 73.43.Nq, 42.50.Dv
Keywords: heavy-ion collisions, quark-gluon plasma, particle production, Monte-Carlo simulation, multiplicity distribution, phase transition, Ginzburg-Landau model

1. Introduction

It is well-known that high energy heavy-ion collision is the main way to study possible creation of a hot new matter state, quark-gluon plasma (QGP) in laboratory. In the collisions, QGP might be formed and the system will be cooling with expanding and undergo a phase transition from deconfined QGP to confined hadrons. Since only the final particles in the collisions are observable experimentally one can search for signals about the phase transition from only those particle.

The order of the phase transition is one of the basic thermodynamic characteristics. A phase transition is said to be of first-order if there is at least one finite gap in the first derivatives of a suitable thermodynamic potential. A transition is said to be of second-order if there is a power-like singularity in at least one of the second derivatives of the potential.

As shown in [1-4] the Ginzburg-Landau (GL) model of phase transitions can be used for investigation of the QGP → hadrons phase transition in the heavy-ion collisions. The parameters of the model have been found for both second-order [1, 2], and first-order [3,4] phase transitions and the scaled factorial moments and a universal scaling exponent ν were calculated. It is suggested that the exponent ν can be used as a helpful diagnostic tool to detect the formation of QGP. As shown [3] for the first-order phase transitions, the value of scaling exponent ν depend on the values of the parameters of the GL model. These parameters can be determined by fitting the experimental multiplicity distribution of charged particles in heavy-ion collisions.

Monte Carlo simulations on intermittency without phase transition for *pp* collisions give results different from theoretical predictions based on the GL model [5].

We study the difference between the scaling exponent ν calculated with the help of GL model of QGP →hadrons phase transitions and that calculated from Monte-Carlo simulations. For Monte-Carlo modeling we use the HIJING Monte-Carlo generator. The HIJING model [6, 7] is the most popular Monte-Carlo program to study particle production in high energy pp, pA and AA collisions. It includes soft and hard interactions, nuclear modification of structure functions, jet quenching, a true geometry of nuclear collisions.

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2. Ginzburg-Landau model for QGP→ hadrons phase transition

The hadron multiplicity distribution for GL theory is given by the functional integral [1]

$$P_n = Z^{-1} \int D\psi P_n^0 e^{-F[\psi]}, \tag{1}$$

were P_0 is the initial distribution for pure state ϕ_0 in which the potential $F[\psi]$ has the minimum,

$$Z = \int D\psi e^{-F[\psi]}, \tag{2}$$

and

$$F[\psi] = \int dz \left[\alpha|\partial\psi(z)/\partial z|^2 + \beta|\psi(z)|^2 + \gamma|\psi(z)|^4 + \delta|\psi(z)|^6\right] \tag{3}$$

stands for the Ginzburg-Landau free energy, where ψ is complex order parameter normalized on the average multiplicity of hadrons in the volume V.

As shown in [1, 3], the simplest form of the initial distribution P_n^0 is the Poissonian distribution

$$P_n^0 = \frac{1}{n!} \exp\left\{-\int\limits_V |\psi(z)|^2 dz\right\} \left(\int\limits_V |\psi(z)|^2 dz\right)^n. \tag{4}$$

Taking into account the probability density P_n^0 we obtain expression for the hadron multiplicity distribution

$$P_n = Z^{-1} \int D\psi \frac{1}{n!} \left(\int\limits_V |\psi(z)|^2 dz\right)^n \exp\left\{-\int\limits_V |\psi(z)|^2 dz\right\} e^{-F[\psi]}. \tag{5}$$

To make the functional integration in (5) is difficult task. We consider a simple case of uniform ψ that is equivalent to setting $\alpha = 0$ in (3). Then the functional integral can be given in the form

$$P_n = Z^{-1} \int_0^\infty dx \frac{(ax)^n e^{-ax}}{n!} e^{-x^3+bx^2+cx}, \quad Z = \int_0^\infty dx e^{-x^3+bx^2+cx} \tag{6}$$

were $x \sim |\psi|^2$

We can determine the constants a, b, c by fitting the experimental data. Parameters of the GL model for first order phase transitions at $(Au + Au)$ collisions were determined in [8]. Result of fitting the experimental data with parameters $a = 72.82$, $b = 7.162$, $c = -11.82$ are plotted on the Fig. 1.

The effective method to study the multiplicity fluctuations consists in examining dependence of the normalized factorial moments F_q on the bin width δ in phase space z [5]

$$F_q = \frac{\langle n(n-1)\cdots(n-q+1)\rangle}{\langle n\rangle^q} = \frac{f_q}{f_1^q}, \tag{7}$$

where $f_q = \langle n(n-1)\cdots(n-q+1)\rangle$, n is the number of hadrons detected in δ in an event, and the average is taken over all events. The multiplicity fluctuations can exhibit intermittency behavior which is described by power-law behavior of F_q on δ

$$F_q \propto \delta^{-\varphi_q}, \tag{8}$$

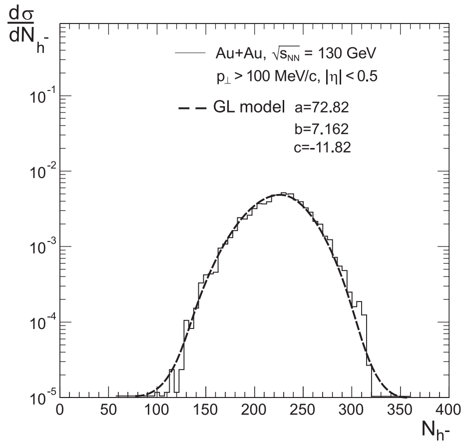

FIG. 1. Comparison of GL model at parameters $a = 72.82,\ b = 7.162,\ c = -11.82$ (dashed line) and experimental data(solid line) for multiplicity distributions of charged particles for heavy-ion $(Au + Au)$ collisions.

where φ_q is referred to as the intermittency index. Besides may use another possibility for the scaling law

$$F_q \propto {F_2}^{\beta_q} \quad , \tag{9}$$

which is found [1-4] to be valid for intermittent systems not possessing the behavior (8). Expression (9) describes the relationship between F_q and F_2, irrespectively of their own dependence on δ. If such property exists, the exponent $\beta_q \propto \frac{\ln F_q}{\ln F_2}$ should be approximately independent on δ.

From (6) and (7) we obtain

$$F_q = \delta^{q-1} \frac{\int_0^\infty dx\,(ax)^{2q}\, e^{-x^3+bx^2+c}}{\int_0^\infty dx\, e^{-x^3+bx^2+cx}} . \tag{10}$$

The β_q exponents can be fitted by

$$\beta_q = (q-1)^\nu \ . \tag{11}$$

The value ν is calculated for parameters $a = 72.82,\ b = 7.162,\ c = -11.82$ of potential (3) and equals to $\nu = 1.35 \pm 0.02$.

3. The Monte-Carlo simulation of the hight energy $(Au + Au)$ collisions

Let is calculate the value of the scaling exponent ν with the help of the HIJING Monte-Carlo generator. The method used to analyze the intermittent behaviour of charge particle production is the study of scaled factorial moments F_q, described in terms of pseudo-rapidity η, where η is related to the spatial emission angle θ by the relation

$$\eta = -\ln(tg\frac{\theta}{2}) \ . \tag{12}$$

The phase-space interval $\Delta\eta$ is divided in M bins of width $\delta\eta = \Delta\eta/M$. The scaled factorial moments are defined as

$$F_q = N^{q-1} \left(\frac{\left\langle \sum\limits_{i=1}^{N} n_i(n_i-1)...(n_i-q+1) \right\rangle}{\left(\left\langle \sum\limits_{i=1}^{N} n_i \right\rangle \right)^q} \right) \tag{13}$$

where n_i is the number of particles in the i-th bin, running from 1 to M, and $< n >$ is the average multiplicity in the whole $\Delta\eta$ interval. For a given order q in the pseudo-rapidity space, the F_q moments are normalized over all events. The results of the Monte-Carlo simulation of the central $Au + Au$ collisions at $\sqrt{S_{NN}} = 130GeV$ are plotted on Fig.2−5. The value of the scaling exponent $\nu = 1.3822$.

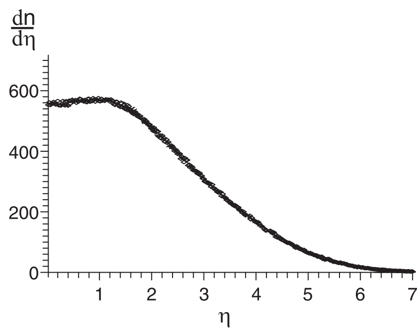

FIG. 2. The rapidity distribution pseudo-rapidity distribution $\frac{dn}{d\eta}$ of charge particles in the central $Au + Au$ collisions at $\sqrt{S_{NN}} = 130GeV$

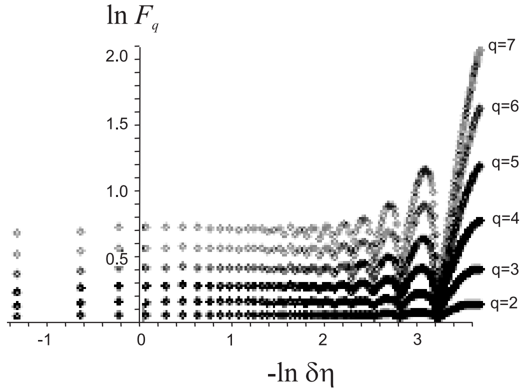

FIG. 3. The variation of $\ln F_q$ as a function of $-\ln \delta\eta$ for q=2,3,4,5,6,7.

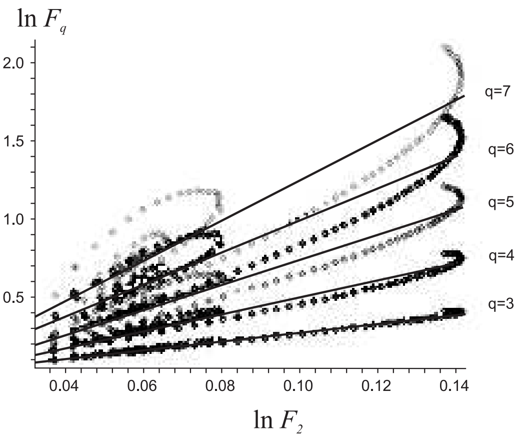

FIG. 4. $\ln F_q$ vs $\ln F_2$ for various value of q = 3,4,5,6,7

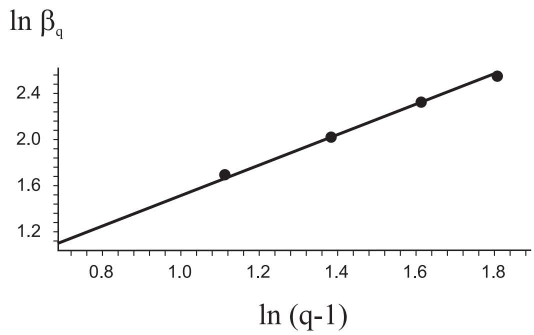

FIG. 5. $\ln \beta_q$ vs $\ln(q-1)$. Dots are determined from Fig. 4; the solid line is a fit.

4. Conclusion

We have studied the multiplicity fluctuations as phenomenological manifestation of QGP→hadron phase transition in the framework both of the GL model for the first-order phase transition and of the

Monte-Carlo simulation by means of HIJING Mote-Carlo generator. We have fitted the multiplicity distribution of charged particles of the $(Au + Au)$ collisions by means of the GL model and found the values of its parameters. We compare the values of the scaling exponents ν for both cases, it is shown that the value of the ν for the Monte-Carlo simulation is different from that calculated with the help of GL model.

Acknowledgments

This research was supported in part by the BFFI-Grant F06R-209 (Minsk, Belarus).

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