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# Manifestation of tensor polarizabilities of the deuteron in storage-ring experiments

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The tensor electric and magnetic polarizabilities of the deuteron give important information about spin-dependent nuclear forces. If a resonant horizontal electric field acts on a deuteron beam circulating into a storage ring, the tensor electric polarizability stimulates the buildup of the vertical polarization of the deuteron (the Baryshevsky effect). The tensor magnetic polarizability of the deuteron causes the spin rotation with two frequencies and experiences beating. General formulas describing these effects have been derived. The problem of the influence of tensor polarizabilities on spin dynamics in a deuteron electric-dipole-moment experiment in storage rings has been investigated. Doubling the resonant frequency used in this experiment dramatically amplifies the Baryshevsky effect and provides the opportunity to make high-precision measurements of the deuteron's tensor electric polarizability. The tensor magnetic polarizability of the deuteron can be measured with the initially tensor polarized beam that can acquire a final horizontal vector polarization of order of 1%.

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## 1. Introduction

Electric and magnetic polarizabilities are important properties of deuteron and other nuclei. Tensor electric and magnetic polarizabilities are defined by spin interactions of nucleons. Measurement of these quantities gives an important information about an interaction between spins of nucleons and provides a good possibility to examine the theory of spin-dependent nuclear forces.

The methods of determination of these important electromagnetic properties of deuteron have been proposed by V. Baryshevsky *at al.* [1–3]. If an electric field acts on a deuteron beam circulating into a storage ring, the presence of the tensor electric polarizability leads to the appearance of an interaction quadratic in spin. When the electric field in the particle's rest frame oscillates at the resonant frequency, the effect similar to the nuclear magnetic resonance takes place. This effect stimulates the buildup of the vertical polarization (BVP) of deuteron beam [1–3]. Another effect defined by the tensor magnetic polarizability of deuteron consists in the spin rotation in the horizontal plane at two frequencies instead of expected rotation at the g−2 frequency [2, 3]. In Refs. [1–3], the approach based on equations defining dynamics of polarization vector and polarization tensor has been used. To check obtained results and develop a more general theory, we follow the quite different method of spin amplitudes (see Refs. [4, 5]) which is partially changed. We derive general formulae describing the BVP caused by the tensor electric polarizability of deuteron in storage rings (the Baryshevsky effect). We propose to use the initially tensor polarized deuteron beam that final vector polarization in the

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horizontal plane conditioned by the tensor magnetic polarizability can be measurable.

## 2. Hamiltonian approach in the method of spin amplitudes

The method of spin amplitudes uses quantum mechanics formalism to more easily describe spin dynamics (see Refs. [4, 5]). Vector and tensor polarization of particles/nuclei with spin $S \geq 1$ are specified by the polarization vector  and the polarization tensor $P_{ij}$ which are given by [6]

$$P_i = \frac{< S_i >}{S}, \quad P_{ij} = \frac{3 < S_i S_j + S_j S_i > -2S(S+1)\delta_{ij}}{2S(2S-1)}, \quad i,j = x,y,z, \tag{1}$$

where $S_i$ are corresponding spin matrices and $S$ is the spin quantum number. The polarization tensor satisfies the conditions $P_{ij} = P_{ji}$ and $P_{xx} + P_{yy} + P_{zz} = 1$ and therefore has five independent components. Additional tensors composed of products of three or more spin matrices are needed only for the exhaustive description of polarization of particles/nuclei with spin $S \geq 3/2$.

The spin matrices for spin-1 particles have the form

$$S_x = \tfrac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \tfrac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{2}$$

The nontrivial spin dynamics predicted in Refs. [1–3] and conditioned by the tensor electric and magnetic polarizabilities of the deuteron is a good example of importance of spin tensor interactions in the physics of polarized beams in storage rings. Tensor interactions of deuteron can also be described with the method of spin amplitudes. In this case, three-component spinors and 3×3 matrices should be used. The method of spin amplitudes is mathematically advantageous because transporting the three-component spinor is much simpler than transporting the three-dimensional polarization vector $P$ and five independent components of the polarization tensor $P_{ij}$ together.

When the deuteron's spin projection onto the direction defined by the spherical angles $\theta, \psi$ is equal to unit ($\lambda = 1$), the components of the polarization vector and the polarization tensor are given by (see, e.g., Ref. [7])

$$P = \begin{pmatrix} \sin\theta\cos\psi \\ \sin\theta\sin\psi \\ \cos\theta \end{pmatrix}, \quad P_{ij} = \frac{3}{2}\begin{pmatrix} \sin^2\theta\cos^2\psi - \frac{1}{3} & \sin^2\theta\sin\psi\cos\psi & \sin\theta\cos\theta\cos\psi \\ \sin^2\theta\sin\psi\cos\psi & \sin^2\theta\sin^2\psi - \frac{1}{3} & \sin\theta\cos\theta\sin\psi \\ \sin\theta\cos\theta\cos\psi & \sin\theta\cos\theta\sin\psi & \cos^2\theta - \frac{1}{3} \end{pmatrix}. \tag{3}$$

When $\lambda = 0$,

$$P = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad P_{ij} = -3\begin{pmatrix} \sin^2\theta\cos^2\psi - \frac{1}{3} & \sin^2\theta\sin\psi\cos\psi & \sin\theta\cos\theta\cos\psi \\ \sin^2\theta\sin\psi\cos\psi & \sin^2\theta\sin^2\psi - \frac{1}{3} & \sin\theta\cos\theta\sin\psi \\ \sin\theta\cos\theta\cos\psi & \sin\theta\cos\theta\sin\psi & \cos^2\theta - \frac{1}{3} \end{pmatrix}. \tag{4}$$

We follow the traditional quantum mechanical approach perfectly expounded by R. Feynman [8] and use the matrix Hamilton equation and the matrix Hamiltonian $H$ for determining an evolution of the spin wave function:

$$i\tfrac{d\Psi}{dt} = H\Psi, \quad \Psi = \begin{pmatrix} C_1(t) \\ C_0(t) \\ C_{-1}(t) \end{pmatrix}, \tag{5}$$

where $H$ is $3 \times 3$ matrix, $\Psi$ is the three-component spin wave function (spinor), $H_{ij} = H^*_{ji}$ and $i, j = 1, 0, -1$.

A determination of spin dynamics can be divided into several stages, namely i) a solution of Hamilton equation (5) and a determination of eigenvalues and eigenvectors of the Hamilton matrix $H$; ii) a derivation of spin wave function consisting in a solution of a set of three linear algebraic equations; iii) a calculation of time evolution of polarization vector and polarization tensor.

## 3. Hamilton operator in a cylindrical coordinate system

The spin dynamics can be analytically calculated when a storage ring is either circular or divided into circular sectors by empty spaces. In this case, the use of cylindrical coordinates can be very successful. Equation of spin motion in storage rings in a cylindrical coordinate system has the form [9]

$$\begin{aligned}\frac{dS}{dt} = \omega_a \times S, \quad \omega_a = -\frac{e}{m}\Big\{aB - \frac{a\gamma}{\gamma+1}\beta(\beta\cdot B) + \left(\frac{1}{\gamma^2-1} - a\right)(\beta \times E) \\ +\frac{1}{\gamma}\left[B_\parallel - \frac{1}{\beta^2}\left(\beta \times E\right)_\parallel\right] + \frac{\eta}{2}\left(E - \frac{\gamma}{\gamma+1}\beta(\beta\cdot E) + \beta\times B\right)\Big\},\end{aligned} \quad (6)$$

where $a = (g-2)/2$, $g = 2\mu m/(eS)$, $\eta = 2dm/(eS)$, and $d$ is the electric dipole moment (EDM). The sign $\|$ means a horizontal projection for any vector. We do not consider effects caused by perturbations of particle trajectory investigated in Ref. [9]. The equation of spin motion in the rotating frame coincides with that in the cylindrical coordinate system because the horizontal axis of this system rotates at the instantaneous angular frequency of orbital revolution.

The Hamiltonian in the rotating frame has the form

$$\mathcal{H} = \mathcal{H}_0 + S \cdot \omega_a, \quad (7)$$

where $\omega_a$ is defined by Eq. (6).

The particle in the rotating frame is localized and ideally is in rest. Therefore, we can direct the $x$- and $y$-axes in this frame along the radial and longitudinal axes, respectively. This procedure is commonly used (see Refs. [4–6]) and results in the direct substitution of spin matrices (2) for $S_\rho$ and $S_\phi$:

$$S_\rho = S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_\phi = S_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (8)$$

The matrix $S_z$ remains unchanged.

## 4. Corrections to the Hamilton operator for tensor polarizabilities of the deuteron

Corrections to the Hamilton operator for deuteron polarizabilities contain scalar and tensor parts. The scalar part is spin-independent and can be disregarded.

The interaction Hamiltonian depending on the electric and magnetic polarizabilities is given by

$$V = V_e + V_m = -\frac{1}{2}\alpha_{ik}E'_iE'_k - \frac{1}{2}\beta_{ik}B'_iB'_k, \quad (9)$$

where $\alpha_{ik}$ and $\beta_{ik}$ are the tensors of electric and magnetic polarizabilities, $E'$ and $B'$ are fields in the particle's rest frame, i.e., in the rotating frame. In this frame, all quantities are marked by primes.

The correction to the Hamilton operator in the rotating frame is equal to

$$V = -\frac{1}{2\gamma}\left(d' \cdot E' + m' \cdot B'\right) = -\frac{\alpha_T}{\gamma}(S \cdot E')^2 - \frac{\beta_T}{\gamma}(S \cdot B')^2, \tag{10}$$

where $\alpha_T$ and $\beta_T$ are the tensor electric and magnetic polarizabilities, respectively. The modulation of normalized velocity can be given by (see Ref. [10])

$$\beta = \frac{p}{\sqrt{m^2+p^2}} = \beta_0 + \Delta\beta_0 \cdot \cos(\omega t + \varphi) e_\phi, \quad \beta_0 = \frac{p_0}{\sqrt{m^2+p_0^2}}, \quad \gamma_0 = \frac{\sqrt{m^2+p_0^2}}{m}. \tag{11}$$

Owing to this modulation, the radial electric field in the particle's rest frame has the oscillatory part. The effect of the modulation on the BVP is described by the last term in Eq. (6) proportional to $\beta \times B$.

If we retain only first-order terms in $\Delta\beta_0$, the particle momentum is defined by the equation

$$p = \frac{m\beta}{\sqrt{1-\beta^2}} = p_0 + \gamma_0^3 m \Delta\beta_0 \cdot \cos(\omega t + \varphi) e_\phi. \tag{12}$$

Eq. (10) can be transformed to the form

$$V = -\frac{\alpha_T}{\gamma}(\beta\gamma B_z S_\rho + E_\phi S_\phi)^2 - \beta_T \gamma B_z^2 S_z^2. \tag{13}$$

The term proportional to magnetic field $B_z$ is much bigger for the deuteron. To simplify the calculation, we neglect the effect of the longitudinal electric field and use the approximation

$$V = -\gamma B_z^2(\alpha_T \beta^2 S_\rho^2 + \beta_T S_z^2). \tag{14}$$

The quantities $\gamma$ and $\beta^2\gamma$ can be expanded in series of $\Delta\beta_0$:

$$\begin{gathered}\gamma = \gamma_0 + \beta_0\gamma_0^3 \cdot \Delta\beta_0 \cos(\omega t + \varphi) + \tfrac{1}{4}(1 + 3\beta_0^2\gamma_0^2)\gamma_0^3(\Delta\beta_0)^2 \left\{1 + \cos\left[2(\omega t + \varphi)\right]\right\}, \\ \beta^2\gamma = \beta_0^2\gamma_0 + (2 + \beta_0^2\gamma_0^2)\beta_0\gamma_0 \cdot \Delta\beta_0 \cos(\omega t + \varphi) \\ + \tfrac{1}{4}(2 + 5\beta_0^2\gamma_0^2 + 3\beta_0^4\gamma_0^4)\gamma_0(\Delta\beta_0)^2 \left\{1 + \cos\left[2(\omega t + \varphi)\right]\right\}.\end{gathered} \tag{15}$$

Eqs. (14),(15) define the corrections to the Hamilton operator for the tensor polarizabilities of deuteron.

## 5. Solution of matrix Hamilton equation

Matrix Hamiltonian (5) takes the form

$$H = \begin{pmatrix} E_0 + \omega_0 + \mathcal{A} + \mathcal{B} & 0 & \mathcal{A} \\ 0 & E_0 + 2\mathcal{A} & 0 \\ \mathcal{A} & 0 & E_0 - \omega_0 + \mathcal{A} + \mathcal{B} \end{pmatrix}, \tag{16}$$

where

$$\begin{gathered}\mathcal{A} = a_0 + a_1\cos(\omega t + \varphi) + a_2\cos\left[2(\omega t + \varphi)\right], \quad \mathcal{B} = b_0 + b_1\cos(\omega t + \varphi) + b_2\cos\left[2(\omega t + \varphi)\right], \\ a_0 = -\tfrac{1}{2}\alpha_T B_z^2\gamma_0\left[\beta_0^2 + \tfrac{1}{4}(2 + 5\beta_0^2\gamma_0^2 + 3\beta_0^4\gamma_0^4)(\Delta\beta_0)^2\right], \quad a_1 = -\tfrac{1}{2}\alpha_T B_z^2(2 + \beta_0^2\gamma_0^2)\beta_0\gamma_0 \cdot \Delta\beta_0, \\ a_2 = -\tfrac{1}{8}\alpha_T B_z^2(2 + 5\beta_0^2\gamma_0^2 + 3\beta_0^4\gamma_0^4)\gamma_0(\Delta\beta_0)^2, \quad b_0 = -\beta_T B_z^2\gamma_0\left[1 + \tfrac{1}{4}(1 + 3\beta_0^2\gamma_0^2)\gamma_0^2(\Delta\beta_0)^2\right], \\ b_1 = -\beta_T B_z^2\beta_0\gamma_0^3 \cdot \Delta\beta_0, \quad b_2 = -\tfrac{1}{4}\beta_T B_z^2(1 + 3\beta_0^2\gamma_0^2)\gamma_0^3(\Delta\beta_0)^2,\end{gathered} \tag{17}$$

and $E_0$ is the zero energy level. In Hamiltonian (16), the EDM effect is not taken into account.

We consider the spin dynamics near a resonance. Zero component of spin is not mixed with other components. The solution of Eq. (16) has the form

$$\begin{gathered}D_1(t)=\left[\cos{(\omega' t)}-i\frac{\omega_0-\omega}{\omega'}\sin{(\omega' t)}\right]D_1(0)-i\frac{\mathcal{E}}{\omega'}\sin{(\omega' t)}D_{-1}(0),\\ D_{-1}(t)=-i\frac{\mathcal{E}^*}{\omega'}\sin{(\omega' t)}D_1(0)+\left[\cos{(\omega' t)}+i\frac{\omega_0-\omega}{\omega'}\sin{(\omega' t)}\right]D_{-1}(0),\\ \omega'=\sqrt{(\omega_0-\omega)^2+\mathcal{E}\mathcal{E}^*},\quad \mathcal{E}=\frac{a_2}{2}\exp{(-2i\varphi)}.\end{gathered}$$

The initial spin amplitudes take the form

$$\begin{gathered}C_1(t)=\exp\left\{-i\left[(E_0+\omega+a_0+b_0)t+\frac{a_1+b_1}{\omega}f(t)+\frac{a_2+b_2}{2\omega}g(t)\right]\right\}D_1(t),\quad C_1(0)=D_1(0),\\ C_{-1}(t)=\exp\left\{-i\left[(E_0-\omega+a_0+b_0)t+\frac{a_1+b_1}{\omega}f(t)+\frac{a_2+b_2}{2\omega}g(t)\right]\right\}D_{-1}(t),\quad C_{-1}(0)=D_{-1}(0).\end{gathered}\tag{18}$$

The resonance at the doubled frequency $\omega\approx 2\omega_0$ can be investigated in a similar way. The evolution of the spin amplitudes is given by

$$\begin{gathered}C_1(t)=\exp\left\{-i\left[(E_0+\frac{\omega}{2}+a_0+b_0)t+\frac{a_1+b_1}{\omega}f(t)+\frac{a_2+b_2}{2\omega}g(t)\right]\right\}D_1(t),\quad C_1(0)=D_1(0),\\ C_0(t)=\exp\left\{-i\left[(E_0+2a_0)t+\frac{2a_1}{\omega}f(t)+\frac{a_2}{\omega}g(t)\right]\right\}C_0(0),\\ C_{-1}(t)=\exp\left\{-i\left[(E_0-\frac{\omega}{2}+a_0+b_0)t+\frac{a_1+b_1}{\omega}f(t)+\frac{a_2+b_2}{2\omega}g(t)\right]\right\}D_{-1}(t),\quad C_{-1}(0)=D_{-1}(0),\end{gathered}\tag{19}$$

where

$$\begin{gathered}D_1(t)=\left(\cos\frac{\omega'' t}{2}-i\frac{2\omega_0-\omega}{\omega''}\sin\frac{\omega'' t}{2}\right)D_1(0)-i\frac{2\mathcal{E}'}{\omega''}\sin\frac{\omega'' t}{2}D_{-1}(0),\\ D_{-1}(t)=-i\frac{2\mathcal{E}'^*}{\omega''}\sin\frac{\omega'' t}{2}D_1(0)+\left(\cos\frac{\omega'' t}{2}+i\frac{2\omega_0-\omega}{\omega''}\sin\frac{\omega'' t}{2}\right)D_{-1}(0),\\ \mathcal{E}'=\frac{a_1}{2}\exp{(-i\varphi)},\quad \omega''=\sqrt{(2\omega_0-\omega)^2+4\mathcal{E}'\mathcal{E}'^*}.\end{gathered}$$

## 6. Spin dynamics caused by tensor electric polarizability of the deuteron

For spin-1 particles, three components of polarization vector and related components of polarization tensor are

$$\begin{gathered}P_\rho=\frac{1}{\sqrt{2}}(C_1C_0^*+C_1^*C_0+C_0C_{-1}^*+C_0^*C_{-1}),\ P_\phi=\frac{i}{\sqrt{2}}(C_1C_0^*-C_1^*C_0+C_0C_{-1}^*-C_0^*C_{-1}),\\ P_z=(C_1C_1^*-C_{-1}C_{-1}^*),\quad P_{\rho\rho}=\frac{3}{2}(C_1C_{-1}^*+C_1^*C_{-1}+C_0C_0^*)-\frac{1}{2},\\ P_{\phi\phi}=-\frac{3}{2}(C_1C_{-1}^*+C_1^*C_{-1}-C_0C_0^*)-\frac{1}{2},\quad P_{zz}=C_1C_1^*-2C_0C_0^*+C_{-1}C_{-1}^*,\\ P_{\rho\phi}=i\frac{3}{2}(C_1C_{-1}^*-C_1^*C_{-1}).\end{gathered}\tag{20}$$

The horizontal components, $P_\rho$ and $P_\phi$, do not give necessary information about the investigated effect because they undergo fast oscillations caused by the g−2 spin precession. The change of the vertical component, $P_z$, is a relatively slow process. When $\omega\approx\omega_0$, the evolution of the vertical component of polarization vector is expressed by

$$\begin{gathered}P_z(t)=\left[1-\frac{\mathcal{E}_0^2}{\omega'^2}\left(1-\cos{(2\omega' t)}\right)\right]P_z(0)\\ +\frac{2\mathcal{E}_0}{3\omega'}\left\{\frac{1}{2}[P_{\rho\rho}(0)-P_{\phi\phi}(0)]\left[\frac{\omega_0-\omega}{\omega'}\cos{(2\varphi)}\Big(1-\cos{(2\omega' t)}\Big)-\sin{(2\varphi)}\sin{(2\omega' t)}\right]\right.\\ \left.+P_{\rho\phi}(0)\left[\frac{\omega_0-\omega}{\omega'}\sin{(2\varphi)}\Big(1-\cos{(2\omega' t)}\Big)+\cos{(2\varphi)}\sin{(2\omega' t)}\right]\right\},\quad \mathcal{E}_0=\frac{a_2}{2},\end{gathered}\tag{21}$$

where the quantities $a_2$ and $\omega'$ are defined by Eqs. (17) and (18), respectively.

For the resonance at the doubled frequency $\omega\approx 2\omega_0$, the evolution of $z$ component of polarization vector is given by

$$\begin{gathered}P_z(t)=\left[1-\frac{4\mathcal{E}_0'^2}{\omega''^2}\left(1-\cos{(\omega'' t)}\right)\right]P_z(0)+\frac{2\mathcal{E}_0'}{3\omega''}\left\{[P_{\rho\rho}(0)-P_{\phi\phi}(0)]\left[\frac{2\omega_0-\omega}{\omega''}\cos{(\varphi)}\Big(1-\cos{(\omega'' t)}\Big)\right.\right.\\ \left.\left.-\sin{(\varphi)}\sin{(\omega'' t)}\right]+2P_{\rho\phi}(0)\left[\frac{2\omega_0-\omega}{\omega''}\sin{(\varphi)}\Big(1-\cos{(\omega'' t)}\Big)+\cos{(\varphi)}\sin{(\omega'' t)}\right]\right\},\quad \mathcal{E}_0'=\frac{a_1}{2},\end{gathered}\tag{22}$$

where the quantities $a_1$ and $\omega''$ are defined by Eqs. (17) and (20), respectively.

Eqs. (17),(21), and (22) show that resonance frequency doubling leads to a dramatic amplification of the Baryshevsky effect. In this case, the EDM effect becomes nonresonant and does not influence the spin dynamics.

## 7. Measurement of tensor electric polarizability of the deuteron in storage-ring experiments

To discover the Baryshevsky effect, it is necessary to stimulate the BVP conditioned by the tensor electric polarizability of deuteron and to avoid a similar effect caused by the magnetic moment. The measurement of the tensor electric polarizability of deuteron in a storage ring needs the field configuration similar to that proposed for the deuteron EDM experiment [10]. However, the resonance frequency should be doubled ($\omega \approx 2\omega_0$). Resonance frequency doubling cannot be implemented in the designed EDM ring. In this ring, the eigenfrequency of free synchrotron oscillations must be chosen close to the g−2 frequency, $\omega_a$, and the resonance effect is created by the beatings between two rf frequencies [10]. Therefore, the measurement of the tensor electric polarizability of deuteron needs another ring or at least rf cavities different from that developed for the deuteron EDM experiment.

The EDM-dependent evolution of deuteron spin in the EDM experiment has been calculated in detail in Ref. [11]. The dynamics of the vertical component of polarization vector is given by

$$P_z^{(EDM)}(t) = \frac{\mathcal{E}_0''}{\Omega'}\left\{\frac{\omega_0-\omega}{\Omega'}\cos(\psi-\varphi)\left[1-\cos(\Omega' t)\right]+\sin(\psi-\varphi)\sin(\Omega' t)\right\}, \qquad (23)$$

where

$$\Omega' = |\Omega'| = \sqrt{(\omega_0-\omega)^2+\mathcal{E}_0''^2}, \quad \mathcal{E}_0'' = -\frac{1}{2}dB_z\cdot\Delta\beta_0\left(1+\frac{a\gamma_0^2\omega}{\omega_0}\right), \qquad (24)$$

and $\psi$ defines the direction of spin at zero time. The initial polarization is supposed to be horizontal. When $\Omega' t \ll 1$,

$$P_z^{(EDM)} = \mathcal{E}_0'' t\sin(\psi-\varphi) = -\frac{1}{2}dB_z\Delta\beta_0\left(1+\frac{a\gamma_0^2\omega}{\omega_0}\right)t\sin(\psi-\varphi). \qquad (25)$$

We can evaluate the expected sensitivity in the measurement of the tensor electric polarizability of deuteron. The sensitivity to the EDM of $d = 1\times10^{-29}$ e·cm corresponds to the accuracy of $\delta\alpha_T = 1.2\times10^{-43}$ cm$^3$ when $\omega \approx 2\omega_0$ and Eqs. (22),(23)–(24) are used. This estimate is based on the values of $\gamma_0 = 1.28$, $\beta_0 = 0.625$, $\Delta v_0 = 3.5\times10^6$ m/s, and $B_z = 3$ T [10]. There are three independent theoretical predictions for the value of tensor electric polarizability of deuteron, namely $\alpha_T = -6.2\times10^{-41}$ cm$^3$ [12], $-6.8\times10^{-41}$ cm$^3$ [13], and $3.2\times10^{-41}$ cm$^3$ [14]. Two first values are very close to each other but they do not agree with the last result.

In all probability, the best sensitivity in the measurement of $\alpha_T$ can be achieved with the use of a tensor polarized deuteron beam. The initial preferential direction of deuteron polarization should be horizontal. When the vector polarization of such a beam is zero, any spin rotation does not occur. Therefore, there are no related systematical errors caused by the radial magnetic field and some other reasons. In the general case, such systematical errors are proportional to a residual vector polarization of the beam. In this case, the preliminary estimate of experimental accuracy is $\delta\alpha_T \sim 10^{-45} \div 10^{-44}$ cm$^3$.

## 8. Tensor magnetic polarizability of the deuteron in the EDM experiment

The spin dynamics in the planned deuteron EDM experiment is affected by the tensor electric and magnetic polarizabilities of the deuteron. Baryshevsky et al. [2, 3] have shown that the tensor magnetic polarizability causes the spin rotation with two frequencies, $\omega_1$ and $\omega_2$, instead of $\omega_0$ and therefore experiences beating with the frequency $\omega_1-\omega_2\approx\beta_T B^2$. This effect makes it possible to measure the tensor magnetic polarizability of the deuteron in storage ring experiments. In this section, an influence of the tensor magnetic polarizability of the deuteron on the spin motion in the EDM experiment is calculated. We take into consideration the EDM and the tensor magnetic polarizability of the deuteron. In this case, the matrix Hamiltonian takes the form

$$H=\begin{pmatrix} E_0+\omega_0+\mathcal{B} & \mathcal{E} & 0\\ \mathcal{E}^* & E_0 & \mathcal{E}\\ 0 & \mathcal{E}^* & E_0-\omega_0+\mathcal{B}\end{pmatrix}, \tag{26}$$

where

$$\begin{gathered}\mathcal{B}=b_0+b_1\cos{(\omega t+\varphi)}+b_2\cos{[2(\omega t+\varphi)]},\quad b_0=-\beta_T\gamma_0B^2,\\ \mathcal{E}=\mathcal{E}_0\exp{[-i(\omega t+\varphi)]},\quad \mathcal{E}_0=\frac{\mathcal{E}''_0}{\sqrt{2}}.\end{gathered} \tag{27}$$

$\mathcal{E}''_0$ is given by Eq. (24).

If the deuteron beam is vector-polarized and the direction of its polarization is defined by the spherical angles $\theta$ and $\psi$, the evolution of three components of polarization vector is given by

$$\begin{aligned}P_\rho(t)=&\sin\theta\cos{(\omega_0t+\psi)}\cos{(b_0t)}-\sin\theta\cos\theta\sin{(\omega_0t+\psi)}\sin{(b_0t)}\\ &+\sqrt{2}[P_{zz}(0)+P_z(0)]\tfrac{\mathcal{E}_0}{\Delta\omega+b_0}\sin\left(\tfrac{\omega_0+\omega+b_0}{2}t+\varphi\right)\sin\tfrac{\Delta\omega+b_0}{2}t\\ &-\sqrt{2}[P_{zz}(0)-P_z(0)]\tfrac{\mathcal{E}_0}{\Delta\omega-b_0}\sin\left(\tfrac{\omega_0+\omega-b_0}{2}t+\varphi\right)\sin\tfrac{\Delta\omega-b_0}{2}t\\ &+\tfrac{1}{\sqrt{2}}\sin^2\theta\left[\tfrac{\mathcal{E}_0}{\Delta\omega-b_0}\sin\left(\tfrac{3\omega_0-\omega+b_0}{2}t+2\psi-\varphi\right)\sin\tfrac{\Delta\omega-b_0}{2}t\right.\\ &\left.-\tfrac{\mathcal{E}_0}{\Delta\omega+b_0}\sin\left(\tfrac{3\omega_0-\omega-b_0}{2}t+2\psi-\varphi\right)\sin\tfrac{\Delta\omega+b_0}{2}t\right],\\ P_\phi(t)=&\sin\theta\sin{(\omega_0t+\psi)}\cos{(b_0t)}+\sin\theta\cos\theta\cos{(\omega_0t+\psi)}\sin{(b_0t)}\\ &-\sqrt{2}[P_{zz}(0)+P_z(0)]\tfrac{\mathcal{E}_0}{\Delta\omega+b_0}\cos\left(\tfrac{\omega_0+\omega+b_0}{2}t+\varphi\right)\sin\tfrac{\Delta\omega+b_0}{2}t\\ &+\sqrt{2}[P_{zz}(0)-P_z(0)]\tfrac{\mathcal{E}_0}{\Delta\omega-b_0}\cos\left(\tfrac{\omega_0+\omega-b_0}{2}t+\varphi\right)\sin\tfrac{\Delta\omega-b_0}{2}t\\ &+\tfrac{1}{\sqrt{2}}\sin^2\theta\left[\tfrac{\mathcal{E}_0}{\Delta\omega+b_0}\sin\left(\tfrac{3\omega_0-\omega-b_0}{2}t+2\psi-\varphi\right)\sin\tfrac{\Delta\omega+b_0}{2}t\right.\\ &\left.-\tfrac{\mathcal{E}_0}{\Delta\omega-b_0}\cos\left(\tfrac{3\omega_0-\omega+b_0}{2}t+2\psi-\varphi\right)\sin\tfrac{\Delta\omega-b_0}{2}t\right],\\ P_z(t)=&P_z(0)+\sqrt{2}\sin\theta(1+\cos\theta)\tfrac{\mathcal{E}_0}{\Delta\omega+b_0}\sin\left(\tfrac{\Delta\omega+b_0}{2}t+\psi-\varphi\right)\sin\tfrac{\Delta\omega+b_0}{2}t\\ &+\sqrt{2}\sin\theta(1-\cos\theta)\tfrac{\mathcal{E}_0}{\Delta\omega-b_0}\sin\left(\tfrac{\Delta\omega-b_0}{2}t+\psi-\varphi\right)\sin\tfrac{\Delta\omega-b_0}{2}t,\end{aligned}$$

where $\Delta\omega=\omega_0-\omega$ and the initial vertical polarization is defined by

$$P_z(0)=\cos\theta,\quad P_{zz}(0)=\tfrac{1}{2}(3\cos^2\theta-1),\quad P_{zz}(0)\pm P_z(0)=-\tfrac{1}{2}(1\pm\cos\theta)(1\mp3\cos\theta). \tag{28}$$

The formula for $P_z(t)$ agrees with Eq. (23).

These equations confirm the conclusion [2, 3] that the tensor magnetic polarizability of the deuteron causes the spin rotation with two frequencies. This effect is rather small but not negligible. According to Refs. [2, 3], the vertical component of the polarization vector experiences the oscillation with two frequencies. Since $b_0\sim10^{-5}$ s$^{-1}$ and the expected duration of measurement $t \sim 10^3$ s [10], $b_0 t \sim 10^{-2}$. Therefore, the effect of the tensor magnetic polarizability on the spin rotation in the horizontal plane can be observed.

We propose to use the tensor-polarized deuteron beam for measuring the tensor magnetic polarizability of the deuteron. If the initial vector polarization of such a beam in zero, the interaction of the magnetic moment of the deuteron cannot lead to the appearance of any vector polarization. Therefore, nonzero vector polarization of the beam can be conditioned by nothing but the tensor interactions. When the projection of the deuteron spin onto the direction defined by the spherical angles $\theta$ and $\psi$ is fixed and is equal to zero, the polarization vector is given by

$$\begin{array}{c} P_\rho(t) = 2\sin\theta\cos\theta\sin(\omega_0 t+\psi)\sin(b_0 t) + \sqrt{2}P_{zz}(0)\frac{\mathcal{E}_0}{\Delta\omega+b_0}\sin\left(\frac{\omega_0+\omega+b_0}{2}t+\varphi\right)\sin\frac{\Delta\omega+b_0}{2}t \\ -\sqrt{2}P_{zz}(0)\frac{\mathcal{E}_0}{\Delta\omega-b_0}\sin\left(\frac{\omega_0+\omega-b_0}{2}t+\varphi\right)\sin\frac{\Delta\omega-b_0}{2}t \\ +\sqrt{2}\sin^2\theta\left[\frac{\mathcal{E}_0}{\Delta\omega+b_0}\sin\left(\frac{3\omega_0-\omega-b_0}{2}t+2\psi-\varphi\right)\sin\frac{\Delta\omega+b_0}{2}t\right. \\ \left.-\frac{\mathcal{E}_0}{\Delta\omega-b_0}\sin\left(\frac{3\omega_0-\omega+b_0}{2}t+2\psi-\varphi\right)\sin\frac{\Delta\omega-b_0}{2}t\right], \\ P_\phi(t) = -2\sin\theta\cos\theta\cos(\omega_0 t+\psi)\sin(b_0 t) - \sqrt{2}P_{zz}(0)\frac{\mathcal{E}_0}{\Delta\omega+b_0}\cos\left(\frac{\omega_0+\omega+b_0}{2}t+\varphi\right)\sin\frac{\Delta\omega+b_0}{2}t \\ +\sqrt{2}P_{zz}(0)\frac{\mathcal{E}_0}{\Delta\omega-b_0}\cos\left(\frac{\omega_0+\omega-b_0}{2}t+\varphi\right)\sin\frac{\Delta\omega-b_0}{2}t \\ +\sqrt{2}\sin^2\theta\left[\frac{\mathcal{E}_0}{\Delta\omega-b_0}\cos\left(\frac{3\omega_0-\omega+b_0}{2}t+2\psi-\varphi\right)\sin\frac{\Delta\omega-b_0}{2}t\right. \\ \left.-\frac{\mathcal{E}_0}{\Delta\omega+b_0}\cos\left(\frac{3\omega_0-\omega-b_0}{2}t+2\psi-\varphi\right)\sin\frac{\Delta\omega+b_0}{2}t\right], \\ P_z(t) = -2\sqrt{2}\sin\theta\cos\theta\frac{\mathcal{E}_0}{\Delta\omega+b_0}\sin\left(\frac{\Delta\omega+b_0}{2}t+\psi-\varphi\right)\sin\frac{\Delta\omega+b_0}{2}t \\ +2\sqrt{2}\sin\theta\cos\theta\frac{\mathcal{E}_0}{\Delta\omega-b_0}\sin\left(\frac{\Delta\omega-b_0}{2}t+\psi-\varphi\right)\sin\frac{\Delta\omega-b_0}{2}t, \\ P_\rho(0) = P_\phi(0) = P_z(0) = 0, \quad P_{zz}(0) = -3\cos^2\theta+1. \end{array}$$

When $|\Delta\omega|t \ll 1, \ |b_0|t \ll 1$, Eq. (29) takes the form

$$\begin{array}{c} P_\rho(t) = 2b_0 t\sin\theta\cos\theta\sin(\omega_0 t+\psi) + \frac{1}{\sqrt{2}}P_{zz}(0)\mathcal{E}_0 b_0 t^2\cos(\omega_0 t+\varphi) \\ -\frac{1}{\sqrt{2}}\mathcal{E}_0 b_0 t^2\sin^2\theta\cos(\omega_0 t+2\psi-\varphi), \\ P_\phi(t) = -2b_0 t\sin\theta\cos\theta\cos(\omega_0 t+\psi) + \frac{1}{\sqrt{2}}P_{zz}(0)\mathcal{E}_0 b_0 t^2\sin(\omega_0 t+\varphi) \\ -\frac{1}{\sqrt{2}}\mathcal{E}_0 b_0 t^2\sin^2\theta\sin(\omega_0 t+2\psi-\varphi), \quad P_z(t) = 0. \end{array}$$

Eq. (29) shows the possibility of measurement of the tensor magnetic polarizability of the deuteron in storage ring experiments because the final vector polarization of the beam can be of order of 1%.

## 9. Differentiation of effects of EDM and tensor electric polarizability in the deuteron EDM experiment

The Baryshevsky effect caused by the tensor electric polarizability of deuteron should be taken into account when performing the deuteron EDM experiment [1–3]. This effect results in the similar BVP and can imitate the presence of the deuteron EDM of order of $d \sim 10^{-29}$ e·cm. An attainment of such an accuracy is planned [10].

The values of $\alpha_T$ found in Refs. [12–14] correspond to the false EDM moments of $|d| = 3\times10^{-29}$, $3\times10^{-29}$, and $2\times10^{-29}$ e·cm, respectively. However, the EDM and Baryshevsky effects have different symmetries, and the use of clockwise and counterclockwise beams makes it possible to cancel the effect of the tensor electric polarizability in the framework of the deuteron EDM experiment. Nevertheless, the existence of this effect should be taken into account.

One can also use other possibilities of separating the EDM and Baryshevsky effects listed below.

1) The spin dynamics caused by first-order interactions (including the EDM effect) and second-order interactions (including the Baryshevsky effect) is defined by the operator equations of spin motion $\frac{dS}{dt} = A\Omega\times S$ and $\frac{dS_i}{dt} = \beta_{ijk}S_jS_k$, respectively. Therefore, the EDM effect reverses the sign when the beam polarization is reversed while the sign of the Baryshevsky effect remains unchanged.

2) Since both the EDM and Baryshevsky effects depend on the difference $\psi - \varphi$, reversing the beam polarization ($\psi \to \psi + \pi$) is equivalent to the transition to the opposite phase ($\varphi \to \varphi + \pi$). Naturally, such a transition is technically simpler. If two measurements of the BVP give the values $P_{z1}$ and $P_{z2}$, the EDM and Baryshevsky effects are characterized by the values $(P_{z1} - P_{z2})/2$ and $(P_{z1} + P_{z2})/2$, respectively. This is valid for paragraphs 1) and 2).

3) In the particle rest frame, the EDM and Baryshevsky effects are linear and quadratic in the electric field, respectively. The experimental dependence can be determined with changing the amplitude of the resonator field.

4) The frequency of BVP caused by the Baryshevsky effect is approximately twice as large as that conditioned by the EDM.

5) The use of tensor polarized deuteron beam even at the angular frequency $\omega \approx \omega_0$ cancels the EDM effect and main systematical errors.

V. Baryshevsky has found [3] that that the derivative of the tensor polarization component $P_{zz}$ does not depend on the tensor electric polarizability and is proportional to the EDM only. When the vector polarization corresponds to $\cos^2\theta = \frac{1}{3}$, $P_{zz}(0) = 0$. In this case, a growth of this component can be easier observed [3]. Our calculation confirms this result. It follows from Eqs. (18),(18),(20) that the tensor electric polarizability does not affect $P_{zz}$. Therefore, the evolution of $P_{zz}$ would be the same if the tensor electric polarizability were equal to zero. Since the evolution of polarization of vector-polarized deuteron beam depends on the EDM rather than on the tensor magnetic polarizability, the change of $P_{zz}$ is proportional to the EDM.

## 10. Summary

The calculations show that the effects found in Refs. [1–3] can be observed in storage rings. All predictions made in these works have been confirmed and detailed calculations of spin dynamics have been carried out. The method based on the use of Hamiltonian approach and spin wave functions is very convenient for such calculations. Calculated formulae agree with the previously obtained results [1–3].

Performing measurements with the use of resonance $\omega \approx 2\omega_0$ and the initial tensor-polarized beam allows to measure the deuteron's tensor electric polarizability with the accuracy of $10^{-45} \div 10^{-44}$ cm$^3$ ($10^{-6} \div 10^{-5}$ fm$^3$). The problem of influence of tensor electric and magnetic polarizabilities on spin dynamics in the storage-ring deuteron EDM experiment has been investigated. The possibilities to differentiate the EDM and Baryshevsky effects have been found. The tensor magnetic polarizability of the deuteron can be measured with the initially tensor polarized beam that can acquire a final horizontal vector polarization of order of 1%. In this case, any resonant field is not used.

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### References

[1] V. Baryshevsky, A. Shirvel, hep-ph/0503214; V. G. Baryshevsky, hep-ph/0504064; STORI 2005 Conference Proceedings, Schriften des Forschungszentrums Jülich, Matter and Materials, Vol. 30 (2005), pp. 227–230.
[2] V. G. Baryshevsky, A. A. Gurinovich, hep-ph/0506135.
[3] V. G. Baryshevsky, hep-ph/0510158, hep-ph/0603191.
[4] E. D. Courant, Bull. Am. Phys. Soc. **7**, 33 (1962); E. D. Courant and R. D. Ruth, BNL Report No. 51270 (1980).
[5] M. G. Minty, F. Zimmermann, *Measurement and Control of Charged Particle Beams* (Springer-Verlag, Berlin, 2003), Chap. 10.
[6] S. R. Mane, Yu. M. Shatunov and K. Yokoya, Rep. Prog. Phys. **68**, 1997 (2005).
[7] A. J. Silenko, Poverkhnost', No. 3, 98 (2007).
[8] R. P. Feynman, R. B. Leighton, M. Sands, *The Feynman Lectures on Physics. V. 2* (Addison-Wesley, Reading, MA, 1963).
[9] A. J. Silenko, Phys. Rev. ST Accel. Beams **9**, 034003 (2006).
[10] Y. F. Orlov, W. M. Morse, and Y. K. Semertzidis, Phys. Rev. Lett. **96**, 214802 (2006).
[11] A. J. Silenko, hep-ph/0604095.
[12] J.-W. Chen, H. W. Grießhammer, M. J. Savage, R. P. Springer, Nucl. Phys. A **644**, 221 (1998).
[13] X. Ji, Y. Li, Phys. Lett. B **591**, 76 (2004).
[14] J. L. Friar and G. L. Payne, Phys. Rev. C **72**, 014004 (2005).