Nonlinear Dynamics and Applications. Vol. 14 (2007) 175 - 180

Two-fermion production by polarized electron-positron annihilation in the frame of Standard theory of electroweak interaction

T.V. Shishkina* and Vasiliy Andreev

Belarusian State University, Minsk, Belarus

The polarized fermion annihilation process was considered in the lowest order of perturbative theory of electroweak interaction in the frame of traceless method (method of basis spinors). Both differential and total cross sections were calculated and analyzed. Precise fermion mass and weak interaction effects contribution into the total and differential cross sections were investigated. The limit of ultra-relativistic particles scattering suitable for ILC experiments is discussed.

PACS numbers: 12.15.-y

Keywords: electron-positron annihilation, polarization, electroweak interaction

1. Introduction

At planned experiments on ILC (International Linear Collider) the first processes for investigation will be processes of two fermion production by electron-positron annihilation:

$$e^- + e^+ \longrightarrow f + \bar{f} \ . \tag{1}$$

In the frame of ILC-program the center-of-mass energies will rise up to a few hundred GeV and even to several TeV and the luminosity will be as high as 500 fb^{-1}. At these experimental conditions for the precise analysis of data we need the theoretical calculations to be of high accuracy (see, for example, [1] – [3]). In this context using of Lorentz-invariant calculation methods without any approximation and including higher order perturbative theory terms becomes extremely important. We also analyse fermion mass contribution because it is very significant in the case when a photon is emitted in electron or positron momentum direction. As well it should be noted that the analysis of particles polarization can present additional possibilities for searching phenomena beyond the Standard model of electroweak interaction predictions. So we can say that the investigation of these processes is a problem of great interest and importance.

In the paper we make calculations of the electroweak interaction process $e^- + e^+ \longrightarrow f + \bar{f}$, consider polarized initial and(or) final fermions, and make some analysis of fermion mass contribution to the cross sections of considered process.

2. Matrix element of the process

In this paper we consider the following process of two-fermion birth:

$$e^-(p_1) + e^+(p_2) \longrightarrow f(k_1) + \bar{f}(k_2) \ , \tag{2}$$

*E-mail: E-mail:shishkina@bsu.by

There are two Feynman diagrams of this process in the lowest order. One of them corresponds to electromagnetic interaction, another — to weak interaction.

We consider p_1 and p_2 (k_1 and k_2) to be four-momenta of initial (final) particles respectively, λ_{p_1} and λ_{p_2} (λ_{k_1} and λ_{k_2}) — polarizations of initial (final) particles respectively.
Using Feynman rules we can obtain matrix element of the process:

$$M = M_\gamma + M_Z \ , \tag{3}$$

where

$$\begin{aligned} & M_\gamma = i \ N_{k_1} N_{k_2} N_{p_1} N_{p_2} e^2 (2\pi)^4 \bar{U}(k_1, \lambda_{k_1}) \ \gamma_\nu \ V(k_2, \lambda_{k_2}) \frac{1}{q^2} \times \\ & \times \ \bar{V}(p_2, \lambda_{p_2}) \ \gamma^\nu U(p_1, \lambda_{p_1}) \ \delta(k_1 + k_2 - p_1 - p_2) \ , \end{aligned} \tag{4}$$

and

$$\begin{aligned} & M_Z = i \ N_{k_1} N_{k_2} N_{p_1} N_{p_2} \ \frac{g^2}{4 \cos^2 \theta_W} \ (2\pi)^4 \bar{U}(k_1, \lambda_{k_1}) \ \gamma^\mu \ (g_V^f + g_A^f \gamma_5) \ \times \\ & \times \ V(k_2, \lambda_{k_2}) \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{m_Z^2}}{q^2 - m_Z^2} \ \bar{V}(p_2, \lambda_{p_2}) \ \gamma^\nu \ (g_V^e + g_A^e \gamma_5) \ \times \\ & \times U(p_1, \lambda_{p_1}) \ \delta(k_1 + k_2 - p_1 - p_2) \ . \end{aligned} \tag{5}$$

Here $N_p = 1/\left((2\pi)^{\frac{3}{2}} \sqrt{2p_0}\right)$, $p_0 = \sqrt{p^2 + m^2}$, $q = p_1 + p_2 = k_1 + k_2$, $g_V = -\frac{1}{2} + 2 \sin^2 \theta_W$, $g_A = -\frac{1}{2}$, θ_W — Wainberg angle, U and V — Dirac bispinors.

As the next step we reduce this matrix element to scalar functions in the frame of method of basis spinors (MBS) [4]. This method allows to present matrix element (not its square) without using traces. It is compact and quick method of cross section calculation. The additional advantage of MBS is the possibility to create quickly working and elegant computer program due to its block structure.

Evaluation of the Feynman amplitude involving fermions is expressed as a sum of terms which have the form

$$\begin{aligned} & \mathcal{M}_{\lambda_p, \lambda_k} \left(p, s_p, \ k, s_k \ ; Q\right) = \\ & = \mathcal{M}_{\lambda_p, \lambda_k} \left(\left[p\right], \left[k\right] ; Q\right) = \bar{u}_{\lambda_p} \left(p, s_p\right) Q \ u_{\lambda_k} \left(k, s_k\right) \ , \end{aligned} \tag{6}$$

where λ_p (λ_k) are polarizations of external particles with four-momentum p (k) and arbitrary polarization vectors $s_p(s_k)$. The operator Q can be expressed as a sum of products of Dirac γ-matrices. In particular, in considered case operator Q is equal to γ_ν for matrix element M_γ, and to $\gamma^\mu(g_V + g_A \gamma_5)$ for M_Z.
The matrix element (6) with Dirac spinors should be expressed in terms of scalar functions formed from spin and momentum four-vectors of Dirac spinors, including p, s_p, k, s_k and operator Q. In frame of MBS the basis spinors connected with vectors of isotropic tetrad have been used.

Here we present the final results of the reduction for the matrix elements (4) and (5).

$$M_\gamma\left(\lambda_{p_1},\lambda_{p_2},\lambda_{k_1},\lambda_{k_2}\right)=(-1)\,8\pi\alpha\;Q_f\left(\delta_{\lambda_{k_1},-\lambda_{k_2}}+\sqrt{\frac{2}{s}}\;m_f\;\delta_{\lambda_{k_1},\lambda_{k_2}}\right)\times$$
$$\times\left(\delta_{\lambda_{p_1},-\lambda_{p_2}}d^1_{\lambda_{p_1},\lambda_k}(\theta)+\delta_{\lambda_{p_1},\lambda_{p_2}}\sqrt{\frac{2}{s}}\;m_e\;d^1_{0,\lambda_k}(\theta)\right)\;, \tag{7}$$
$$M_Z\left(\lambda_{p_1},\lambda_{p_2};\lambda_{k_1},\lambda_{k_2}\right)=2\pi\alpha\,s\,G_Z\times$$
$$\times\left[\left(\left[g^f_V-\lambda_{k_1}\beta_f g^f_A\right]\delta_{\lambda_{k_1},-\lambda_{k_2}}+\delta_{\lambda_{k_1},\lambda_{k_2}}\sqrt{\frac{2}{s}}\;m_f\,g^f_V\right)\times\right.$$
$$\times\left(\delta_{\lambda_{p_1},-\lambda_{p_2}}d^1_{\lambda_{p_1},\nu}(\theta)\left(g^e_V+\lambda_{p_1}\beta_e g^e_A\right)+\delta_{\lambda_{p_1},\lambda_{p_2}}\sqrt{\frac{2}{s}}\;m_e g^e_V\;d^1_{0,\lambda_k}(\theta)\right)+$$
$$\left.+\delta_{\lambda_{p_1},\lambda_{p_2}}\delta_{\lambda_{k_1},\lambda_{k_2}}\frac{\lambda_{p_1}\lambda_{k_1}m_e m_f g^e_A g^f_A}{s}\left(1-\frac{m_Z^2}{s}\right)\right]\;. \tag{8}$$

Here $\alpha=e^2/4\pi$, $G_Z=1/\left(\cos^2\theta_W\sin^2\theta_W\left(s-m_Z^2+\mathrm{i}m_Z\Gamma_Z\right)\right)$, $\beta_e=\sqrt{1-\frac{4m_e}{s}}$, $\beta_f=\sqrt{1-\frac{4m_f}{s}}$, $\lambda_k=\left(\lambda_{k_1}-\lambda_{k_2}\right)/2$, $d^1_{\lambda,\nu}(\theta)$ — small Wigner function with index 1, which defines angular distribution of final fermions, m_Z — Z-boson mass, Γ_Z — Z-boson decay width, Q_f — fermion charge in terms of electron charge.

3. Differential and total cross sections

Here we present the expressions for differential and total cross sections in the center-of-mass system. It was chosen in correspondence with the geometry of planned experiments on linear colliders and gives simple expressions for cross sections. These expressions are convenient for parametrization and calculation.

So the differential cross section in the center-of-mass system can be written as

$$\frac{d\sigma}{d\theta}=\frac{\beta_f}{\beta_e}\;\frac{|R|^2}{16(2\pi)^2 s}, \tag{9}$$

where angle θ is the scattering angle (the angle between the momentum direction of any initial and final particle).

One can see here that $d\sigma/d\theta\sim|R|^2$, because all constants were included into the phase factor which has a simple form in the center-of-mass system.

Hence the total cross section can be written as

$$\sigma=2\pi\int_0^\pi\frac{d\sigma}{d\theta}\sin\theta\;d\theta. \tag{10}$$

Since helicity states are used as a kind of polarization here, the polarization coefficient P is a relative amount of initial and(or) final particles which have definite helicity state(particle's spin is co-directional to its momentum or just vise versa).

We have calculated the differential and total cross sections in the case of massive particles scattering as well as in ultra-relativistic limit $p_i\gg|m_i|$. The analyze of the contribution of fermion masses into cross sections was performed. As an example of fermions here we have considered muons.

We present the result of our calculations on Figs. 1 - 4.

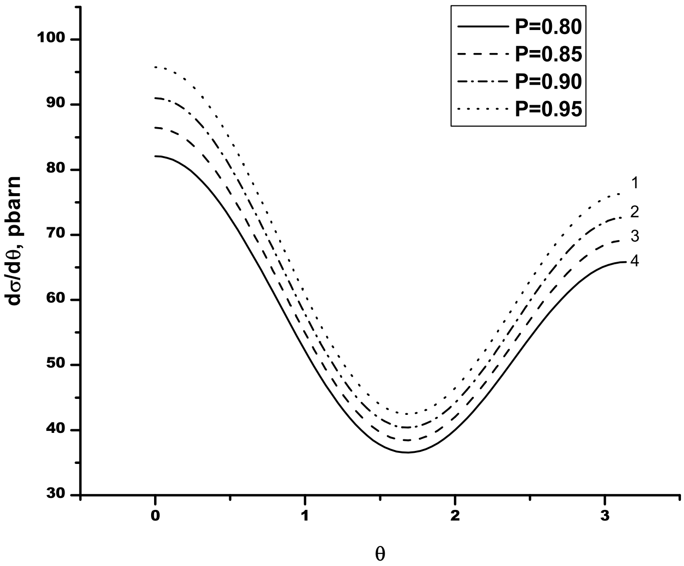

FIG. 1. Differential cross section of the process at interaction energy $E = 400\ GeV$ for different polarization coefficients P.

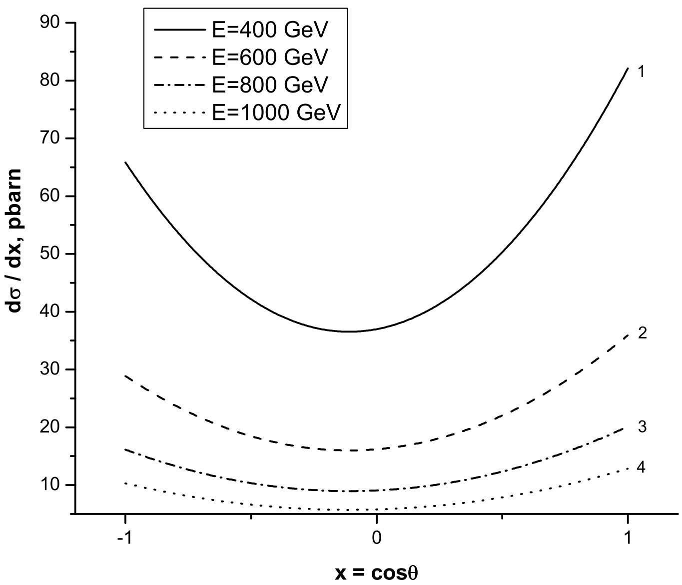

FIG. 2. Differential cross section of the process with $P = 0.8$ for a set of values of interaction energies.

4. Conclusions

So in this paper is we have considered the differential and total cross sections of the electroweak annihilation process $e^- + e^+ \longrightarrow f + \bar{f}$. The case of massive interacting particles and the case of ultra-relativistic limit have been analyzed. Corresponding graphs were constructed. The polarizations of initial and final particles (helicity states) were considered.

Firstly the analysis of fermion masses contribution into differential and total cross sections was performed. As an example of fermions in the final state were taken muons. It was discov-

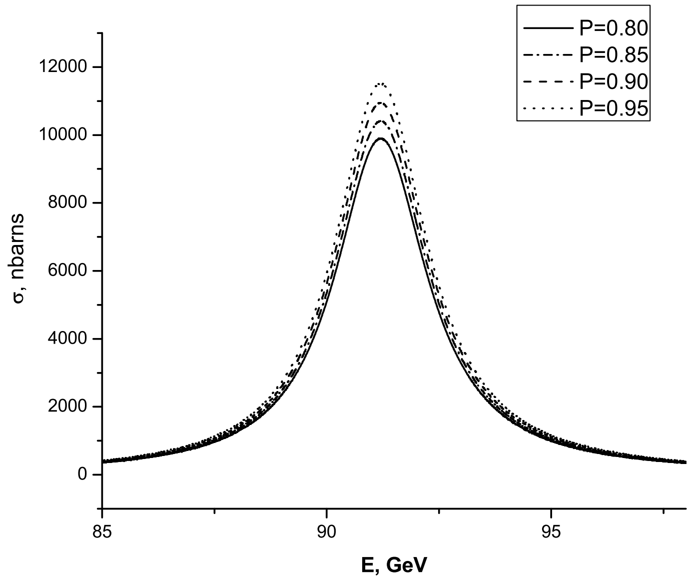

FIG. 3: Total cross section of the process in case of different polarization coefficients P.

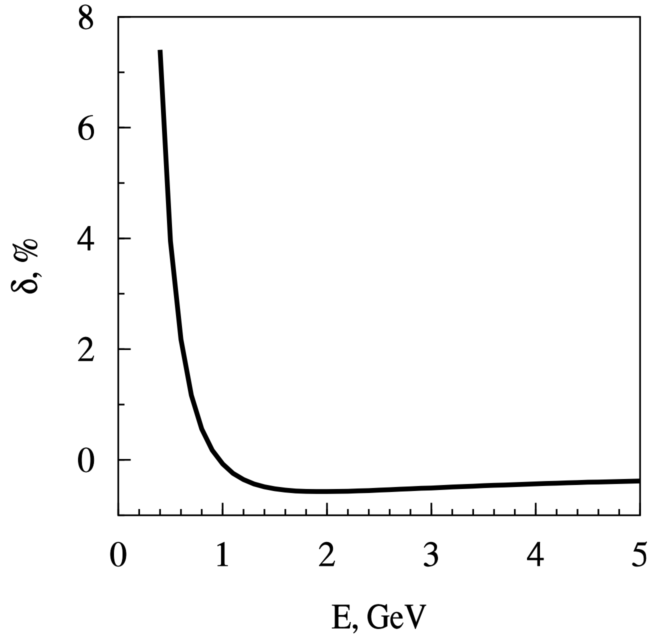

FIG. 4. Relative contribution of fermion masses into the total cross section of the process $e^- + e^+ \longrightarrow \mu^- + \mu^+$ with $P = 0.8$.

ered that this contribution is extremely small as a rule. It is determined mainly by the energy of initial particles $\sqrt{s}$. Contributions into the total cross section have the most considerable value when energy is near $\sqrt{s} = 2m_\mu$ and $\sqrt{s} = M_z$. The relative values of these contributions consist about $1 - 2$ %. With energy increasing this value lowers and amounts to $0.005 - 0.01$ %. For differential cross section the mass contributions are greater at low energies ($\sqrt{s} = 2m_\mu$). The relative value is near 0.1 %. For greater energies this value is negligibly small. The conclusion is that in Standard model in the framework of planned experiments precision $\simeq 1\%$ we need to take into account fermion masses only near Z-boson peak. At greater energies we can use formulae in ultra-relativistic limit.

We have analysed the influence of particles polarization on cross sections of the process (Figs.1-4). One can see that magnitudes of cross sections increase with increasing of polarization coefficient P.

It's evident to achieve results with high precision it is necessary to take into account higher order perturbative theory terms, especially at energies of ILC experiment.

References

[1] T.V. Shishkina. Etudes on theoretical physics. *World Scientific Pub.*, page 213, 2004.

[2] T.V. Shishkina. *Actual Problems on Microworld Phys., Dubna*, page 177, 2007.

[3] I. Marfin T. Shishkina. Elementary partcles and fields. *Nonlinear Phenomena in Complex Systems*, page 710, 2006.

[4] V.V. Andreev. Analytical calculation of feynman amplitudes. *Physics of atomic nuclei*, 66: 383–393, 2003.