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Effect of motion of the reference frame on geometrical form of the line of monopole singularity

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The monopole singularity string, infinite direct line in the rest reference frame, is considered as a rigid physical line, and behavior of its geometrical form under the Lorentz transformations is studied. The method to test the form of any rigid line or surface in relativistical terms is based on the use of light signals emitted from the origin of the reference frame. Influence of the motion of the reference frame on the form of singularity line is as follows: (1) in general case, when the line does not go through the origin of the rest reference frame the line in the moving reference frame becomes a hyperbola; (2) in the special case, when the line of singularity goes through the origin of the rest reference frame, the line in the moving reference preserves its form only modified by the relativistical aberration effect.

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1. Introduction

In the literature concerning magnetic monopoles, the problem of the monopole line of singularity, so-called monopole string, has been considered many times and in many different aspects – we remind only three basic papers by P.A.M. Dirac [1-3]. One may see two opposite viewpoints: (first) the line of monopole singularity is fictitious, coordinate-based, it cannot manifest itself in physical experiments; (second) the line of monopole singularity is quite real and it can be studied in physical experiments.

Certainly, the second point of view may be interpreted as considering the monopole just like a one-dimensional extended object. In other words, the monopole is not a point-like, and thereby it should not be considered as a specific elementary particle. In absence of experimental evidence to existence of monopoles, these two views may be taken as alternative working ideas: each of them should be studied seriously – theoretically and experimentally.

In the present paper, we try to consider the monopole string as a physically real line, remembering that the Dirac monopole potential is not defined at the infinite half-line, whereas the Schwinger's potential is not well defined at the whole infinite line. Then, this singular string is changed by a rigid physical line, and further one can study its geometrical properties with the help of tools of ordinary relativistic kinematics. We investigate how the form of this line will change in dependence of inertial motion of the reference frame. General method to solve the problem is taken from [4]. The method to test the form of any rigid line or surface in

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relativistical terms is based on the use of light signals emitted from the origin of the reference frame.

Influence of the motion of the reference frame on the form of singularity line is:

(1) in general case, when the line does not go through the origin of the rest reference frame the line in the moving reference frame becomes a hyperbola;

(2) in the special case, when the line of singularity goes through the origin of the rest reference frame, the line in the moving reference preserves its form only modified by the relativistical aberration effect.

2. Monopole string as a set of events in the space-time

Let the monopole string be at rest in the reference frame K' and is oriented along the axis z' - -see Fig. 1. Let the monopole string is seen by other inertial observer K:

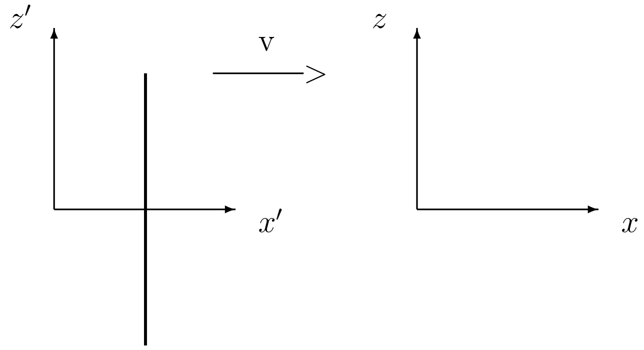

Fig. 1

In the moment of coincidence of two reference frame ($t'_1 = t_1 = 0$) let from the origin light signal be emitted in different directions (event $1'$); which arrives at the moment t'_2 to a point on the line – (event $2'$). This light signal in the reference frame K' is determined by

$$x'(t') = c\ \cos\phi'_2\ t'\ , \qquad z'(t') = c\ \sin\phi'_2\ t'\ ; \tag{1}$$

and the parametric equation for the monopole string is

$$x' = L\ , \qquad z' = \lambda\ , \ \ \lambda \in (-\infty, +\infty)\ . \tag{2}$$

For the event $2'$ one should have two relations

$$x'_2 = c\ \cos\phi'_2\ t'_2 = L\ , \qquad z'_2 = c\ \sin\phi'_2\ t'_2 = \lambda\ \ . \tag{3}$$

Thus, the coordinates of the event $2'$ in terms of L, ϕ'_2 look

$$ct'_2 = \frac{L}{\cos\phi'_2}\ , \qquad x'_2 = c\ \cos\phi'_2\ t'_2 = L\ , \qquad z'_2 = \frac{L\ \sin\phi'_2}{\cos\phi'_2}\ ; \tag{4}$$

With the use of the Lorentz formulas one can establish coordinates of the event $2'$ in the reference frame K (let $V = v/c$ and $ct \Longrightarrow t$)

$$x_2 = \frac{x'_2 - V\ t'_2}{\sqrt{1-V^2}} = \frac{\cos\phi'_2 - V}{\sqrt{1-V^2}}\ t'_2\ ,$$

$$t_2 = \frac{t'_2 - V\ x'_2}{\sqrt{1-V^2}} = \frac{1 - V\ \cos\phi'_2}{\sqrt{1-V^2}}\ t'_2\ ,$$

$$z_2 = z'_2 = \sin\phi_2\ t'_2\ . \tag{5}$$

Taking in mind

$$t'_2 = \frac{\sqrt{1-V^2}}{1 - V\ \cos\phi'_2}\ t_2\ ,$$

one readily produces

$$x_2 = \frac{\cos\phi'_2 - V}{1 - V\ \cos\phi'_2}\ \ t_2 \equiv \cos\phi_2\ t_2\ ,$$
$$z_2 = \sin\phi'_2\ \ \frac{\sqrt{1-V^2}}{1 - V\ \cos\phi'_2}\ \ t_2 \equiv \sin\phi_2\ t_2\ . \tag{6}$$

It is easily verified that introducing the angle variable in (6) is correct Indeed (in fact we check below the invariance of the light velocity under Lorentz transformations)

$$(\cos\phi_2)^2 + (\sin\phi_2)^2 = (\frac{\cos\phi'_2 - V}{1 - V\ \cos\phi'_2})^2 + (\sin\phi'_2\ \ \frac{\sqrt{1-V^2}}{1 - V\ \cos\phi'_2})^2 =$$
$$= \frac{\cos^2\phi'_2 - 2V\cos\phi'_2 + V^2 + \sin^2\phi'_2 - \sin^2\phi'_2 V^2}{(1 - V\ \cos\phi'_2)^2} =$$
$$= \frac{1 - 2V\cos\phi'_2 + \cos^2\phi'_2 V^2}{(1 - V\ \cos\phi'_2)^2} = 1\ .$$

Let us write down the formulas describing relativistic aberration effect (direct and inverse):

$$\cos\phi_2 = \frac{\cos\phi'_2 - V}{1 - V\ \cos\phi'_2}\ , \qquad \sin\phi_2 = \sin\phi'_2\ \ \frac{\sqrt{1-V^2}}{1 - V\ \cos\phi'_2}\ ,$$

$$\cos\phi'_2 = \frac{\cos\phi_2 + V}{1 + V\ \cos\phi_2}\ , \qquad \sin\phi'_2 = \sin\phi_2\ \ \frac{\sqrt{1-V^2}}{1 + V\ \cos\phi_2}\ . \tag{7}$$

3. On geometrical form of the line of singularity in the moving reference frame K

Let start with equation for the monopole string in the rest reference frame K':

$$x' = L\ , \qquad z' = \lambda\ , \qquad \lambda \in (-\infty, +\infty)\ . \tag{8}$$

To this geometrical line one may put in correspondence a special set of events of the type $2'(2)$ in space-time:

$$t'_2 = \frac{L}{\cos\phi'_2}\ , \qquad x'_2 = \cos\phi'_2\ \ t'_2\ , \qquad z'_2 = \sin\phi'_2\ \ t'_2\ . \tag{9}$$

Space-time coordinate of these events may be transformed to the moving reference frame K:

$$t'_2 = \frac{t_2 + Vx_2}{\sqrt{1-V^2}} = \frac{x_2/\cos\phi_2 + Vx_2}{\sqrt{1-V^2}} = \frac{1/\cos\phi_2 + V}{\sqrt{1-V^2}}\ x_2\ ,$$
$$x'_2 = \frac{x_2 + Vt_2}{\sqrt{1-V^2}} = \frac{x_2 + Vx_2/\cos\phi_2}{\sqrt{1-V^2}} = \frac{1 + V/\cos\phi_2}{\sqrt{1-V^2}}\ x_2\ , \qquad z'_2 = z_2 = \lambda\ . \tag{10}$$

Expressing in (8) the variables in terms of new ones (not primed) we get

$$\frac{1+V/\cos\phi_2}{\sqrt{1-V^2}}\, x_2 = L, \qquad z = \lambda \tag{11}$$

Now, taking in mind

$$x_2 = \cos\phi_2\, t_2 \;, \qquad z_2 = \sin\phi_2\, t_2 :$$
$$\frac{1}{\cos\phi_2} = \sqrt{1+\tan^2\phi_2} = \sqrt{1+\frac{z_2^2}{x_2^2}} \;,$$

eq. (11) reads (index 2 is omitted)

$$\frac{1+V\sqrt{1+z^2/x^2}}{\sqrt{1-V^2}}\; x \;= L, \qquad z = \lambda \in (-\infty, +\infty) \;, \tag{12}$$

or with the use of hyperbolical variable

$$(\;\mathrm{ch}\beta + \mathrm{sh}\beta\;\sqrt{1+z^2/x^2}\;)\; x = L \;. \tag{13}$$

Relations (12)–(13) should be considered as describing the form of the monopole string in the moving reference frame.

Now, let us establish geometrical forme of the curve given by (13). Eq. (13) reads

$$(L - \mathrm{ch}\beta\; x)^2 = \mathrm{sh}^2\beta\;(x^2+z^2) \;, \tag{14}$$

and further

$$L^2 - 2Lx\mathrm{ch}\beta \;+ x^2 \;= z^2\;\mathrm{sh}^2\beta \;,$$

or

$$(L^2 - L^2\mathrm{ch}^2\beta) + (L\mathrm{ch}\beta - x)^2 \;= z^2\;\mathrm{sh}^2\beta \;.$$

Evidently, this curve is hyperbola:

$$\frac{(L\mathrm{ch}\beta - x)^2}{L^2\mathrm{sh}^2\beta} \;-\; \frac{z^2\;\mathrm{sh}^2\beta}{L^2\mathrm{sh}^2\beta} = 1 \tag{15}$$

There exists one special case: let $L = 0$, then latter equation becomes equation for a direct line:

$$x^2 - z^2\;\mathrm{sh}^2\beta = 0, \qquad x = \pm\;\mathrm{sh}\beta\; z \;. \tag{16}$$

4. On the forme of the monopole string – the case of arbitrary orientation of the string and velocity vector

Let us generalize the above result to the case of arbitrary orientation of the monopole singular line and velocity vector. We will need an explicit representation for Lorentz formulas with any velocity vector **V** (more details see in the book [12]). With the notation

$$\mathbf{V} = \mathbf{e}\; th\;\beta \;, \qquad \mathbf{e}^2 = 1 \;,$$
$$\frac{1}{\sqrt{1-V^2}} = ch\;\beta, \qquad \frac{V}{\sqrt{1-V^2}} = sh\;\beta, \tag{17}$$

arbitrary Lorentz matrix $L(\mathbf{V}$ is

$$(L_a{}^b) =$$

$$\left| \begin{array}{cccc} ch\,\beta & e_1 sh\,\beta & e_2 sh\,\beta e_2 & e_3 sh\,\beta \\ e_1 sh\,\beta & ch\,\beta - (ch\,\beta - 1)(e_2^2 + e_3^2) & (ch\,\beta - 1)e_1 e_2 & (ch\,\beta - 1)e_2 e_3 \\ e_2 sh\,\beta & (ch\,\beta - 1)e_1 e_2 & ch\,\beta - (ch\,\beta - 1)(e_1^2 + e_3^2) & (ch\,\beta - 1)e_2 e_3 \\ e_3 sh\,\beta & (ch\,\beta - 1)e_1 e_3 & (ch\,\beta - 1)e_2 e_3 & ch\,\beta - (ch\,\beta - 1)(e_1^2 + e_2^2)\ , \end{array} \right|$$

or differently

$$(L_a{}^b) = \left| \begin{array}{cccc} ch\,\beta & e_1 sh\,\beta & e_2 sh\,\beta e_2 & e_3 sh\,\beta \\ e_1 sh\,\beta & 1 + (ch\,\beta - 1)e_1^2 & (ch\,\beta - 1)e_1 e_2 & (ch\,\beta - 1)e_2 e_3 \\ e_2 sh\,\beta & (ch\,\beta - 1)e_1 e_2 & 1 + (ch\,\beta - 1)e_2^2 & (ch\,\beta - 1)e_2 e_3 \\ e_3 sh\,\beta & (ch\,\beta - 1)e_1 e_3 & (ch\,\beta - 1)e_2 e_3 & 1 + (ch\,\beta - 1)e_3^2 \end{array} \right| , \tag{18}$$

or in symbolical form

$$L = \left| \begin{array}{cc} ch\,\beta & \mathbf{e}\ sh\,\beta \\ \mathbf{e}\ sh\,\beta & [\delta_{ij} + (ch\,\beta - 1)e_i e_j] \end{array} \right| . \tag{19}$$

Correspondingly, the Lorentz transformation acts on space-time 4-vector $(t, \mathbf{x})$ as follows

$$t' = ch\,\beta\, t + sh\,\beta\, \mathbf{e}\,\mathbf{x}\ ,$$
$$\mathbf{x}' = \mathbf{e}\ sh\,\beta\, t + \mathbf{x} + (ch\,\beta - 1)\ \mathbf{e}\ (\mathbf{e}\mathbf{x}). \tag{20}$$

Inverse formulas are

$$t = ch\,\beta\, t' - sh\,\beta\, \mathbf{e}\,\mathbf{x}'\ ,$$
$$\mathbf{x} = -\mathbf{e}\ sh\,\beta\, t + \mathbf{x}' + (ch\,\beta - 1)\ \mathbf{e}\ (\mathbf{e}\mathbf{x}')\ . \tag{21}$$

Let in the rest reference frame K' the monopole string is given by the parametric equation

$$\mathbf{x}' = \lambda\, \mathbf{n} + \mathbf{x}_0\ . \tag{22}$$

To this geometrical line in 3-space one may put into correspondence a special set of space-time events $\{\ (t', \mathbf{x}' = (x_1', x_2', x_3')\ \}$ defined as follows:

$$\mathbf{x}' = \mathbf{c}'\, t', \qquad \Longrightarrow \qquad \mathbf{c}'^2 = 1\ , \qquad t' = \sqrt{\mathbf{x}'^2}\ ; \tag{23}$$

that is

$$\left\{\ t' = \sqrt{\mathbf{x}'^2}\ ,\ \mathbf{x}' = \lambda\, \mathbf{n} + \mathbf{x}_0\ \right\}\ . \tag{24}$$

The same events, being viewed from the moving reference frame K, look as

$$\mathbf{x} = \mathbf{c}\, t, \qquad \Longrightarrow \qquad \mathbf{c}^2 = 1\ , \qquad t = \sqrt{\mathbf{x}^2}\ . \tag{25}$$

Thus, the set of events (24) now is described in term of new coordinates as follows:

$$\left\{\ t = \sqrt{\mathbf{x}^2}\ ,\ \mathbf{x} + \mathbf{e}\ [\ sh\,\beta\, t + (ch\,\beta - 1)\ \mathbf{e}\mathbf{x}\ \ ] = \lambda \mathbf{n} + \mathbf{x}_0\ \right\}\ . \tag{26}$$

Now, one should exclude the time-variable:

$$\mathbf{x} + \mathbf{e}\ [\ sh\,\beta\ \sqrt{\mathbf{x}^2} + (ch\,\beta - 1)\ \mathbf{e}\mathbf{x}\ ] = \lambda \mathbf{n} + \mathbf{x}_0\ . \tag{27}$$

This, we have produced equation describing the form the monopole string in the moving reference frame K. In the rest reference frame K' (when $\beta = 0$) eq. (27) represent a direct line.

In the first place, let us verify that the general equation (27) agrees with the particular case studied above. Let it be

$$\mathbf{n} = (0,0,1) \; , \qquad \mathbf{x}_0 = (L,0,0) \; , \qquad \mathbf{e} = (1,0,0) \; ,$$

then eq. (27) becomes

$$x + \text{sh}\, \beta \sqrt{\mathbf{x}^2} + (\text{ch}\, \beta - 1) x = L \; , \qquad y = 0, \qquad z = \lambda \; ;$$

which leads us to the above eq. (14):

$$\text{sh}^2\beta \; (x^2 + z^2) = (L - \text{ch}\, \beta)^2 \; .$$

Now, consider a more general case:

$$\mathbf{n} = (n_1, n_2, n_3) \; , \qquad \mathbf{e} = (1,0,0), \qquad \mathbf{x}_0 = (0,0,0) \; . \tag{28}$$

when a the singularity line goes through the origin $(0,0,0)$ of K'. At this eq. (27) takes the form

$$\begin{aligned} \text{sh}\, \beta \sqrt{x^2 + y^2 + z^2} + x \; \text{ch}\, \beta = \lambda n_1 \; , \\ y = \lambda n_2, \qquad z = \lambda n_3 \; . \end{aligned} \tag{29}$$

in the plane (y, z) one can introduce a rotated coordinate system (Z, Y):

$$z = Z \; \cos\phi = Z \; \frac{n_3}{\sqrt{n_3^2 + n_2^2}}, \qquad y = Z \; \sin\phi = Z \; \frac{n_2}{\sqrt{n_3^2 + n_2^2}} \tag{30}$$

evidently λ looks as

$$\lambda = \frac{y}{n_2} = \frac{z}{n_3} = \frac{Z}{\sqrt{n_3^2 + n_2^2}} \; . \tag{31}$$

Correspondingly, eq. (29) gives

$$\text{sh}\, \beta \sqrt{x^2 + Z^2} + x \; \text{ch}\, \beta = Z\nu, \qquad \nu = \frac{n_1}{\sqrt{n_3^2 + n_2^2}} = \frac{n_1}{\sqrt{1 - n_1^2}} \; . \tag{32}$$

From this it follows

$$x^2 \text{sh}^2\beta + Z^2 \text{sh}^2\beta = Z^2\nu^2 - 2Z\nu \; x\text{ch}\, \beta + x^2 \; \text{ch}^2 \; \beta \; ,$$

or

$$(\nu^2 - \text{sh}^2\beta) \; Z^2 - (\nu \text{ch}\beta) \; 2Zx + x^2 = 0 \; ,$$

that is

$$A = \nu^2 - \text{sh}^2\beta, \quad B = \nu\text{ch}\beta, \qquad AZ^2 - B \; 2Zx + x^2 = 0 \; . \tag{33}$$

Now, in the plane (x, Z) one can perform a special rotation $(x, Z) \Longrightarrow (X'', Z'')$ so that the factor at X'', Z''-term be equal to zero. Let it be

$$x = \cos\phi X'' + \sin\phi Z'', \qquad Z = -\sin\phi X'' + \cos\phi Z'' \; . \tag{34}$$

then eq. (33) becomes

$$A\,[\,\sin^2\phi\,(X'')^2-\sin 2\phi\;X''Z''+\cos^2\phi\,(Z'')^2\,]-$$
$$-B\,[\,-\sin 2\phi\,(X'')^2+2\cos 2\phi\;X''Z''+\sin 2\phi\,(Z'')^2\,]+$$
$$+[\,\cos^2\phi\,(X'')^2+\sin 2\phi\;X''Z''+\sin^2\phi\,(Z'')^2\,]=0\;.$$

From where, after re-grouping the terms we arrive at

$$(A\;\sin^2\phi+\cos^2\phi+B\;\sin 2\phi)(X'')^2+$$
$$+(A\cos^2\phi+\sin^2\phi-B\sin 2\phi)\;(Z'')^2$$
$$-[(A-1)\sin 2\phi+2B\cos 2\phi]X''Z''=0\;. \tag{35}$$

Let us impose the restriction

$$(A-1)\sin 2\phi+2B\cos 2\phi=0, \qquad \Longrightarrow$$
$$\tan 2\phi=\frac{2B}{1-A}=\frac{2\nu\text{ch}\beta}{\text{ch}^2\beta-\nu^2}=\frac{2\nu/\text{ch}\beta}{1-\nu^2/\text{ch}^2\beta}\;.$$

from which it follows

$$\tan\phi=\frac{\nu}{\text{ch}\beta}\;,\qquad \cos^2\phi=\frac{1}{1+\tan^2\phi}=\frac{\text{ch}^2\beta}{\nu^2+\text{ch}^2\beta}\;,\qquad \sin^2\beta=\frac{\nu^2}{\nu^2+\text{ch}^2\beta}\;. \tag{36}$$

Therefore, eq. (35) is much simplified:

$$(A\;\sin^2\phi+\cos^2\phi+B\;\sin 2\phi)(X'')^2+$$
$$+(A\cos^2\phi+\sin^2\phi-B\sin 2\phi)\;(Z'')^2=0\;.$$

Taking in mid two identities:

$$A\;\sin^2\phi+\cos^2\phi+B\;\sin 2\phi=\frac{(\nu^2-\text{sh}^2\beta)\nu^2+\text{ch}^2\beta+2\nu^2\text{ch}^2\beta}{\nu^2+\text{ch}^2\beta}=$$
$$=\frac{(\nu^2+1)(\nu^2+\text{ch}^2\beta)}{\nu^2+\text{ch}^2\beta}=(\nu^2+1)\;,$$
$$(A\cos^2\phi+\sin^2\phi-B\sin 2\phi)=\frac{(\nu^2-\text{sh}^2\beta)\text{ch}^2\beta+\nu^2-2\nu^2\text{ch}^2\beta}{\nu^2+\text{ch}^2\beta}=$$
$$=\frac{\nu^2-\text{sh}^2\beta\text{ch}^2\beta-\nu^2\text{ch}^2\beta}{\nu^2+\text{ch}^2\beta}=-\text{sh}^2\beta\;,$$

the latter relation takes the form of equation for a direct line:

$$(\nu^2+1)(X'')^2-\text{sh}^2\beta(Z'')^2=0\;; \tag{37}$$

remember (see (32))

$$\nu=\frac{n_1}{\sqrt{n_3^2+n_2^2}}=\frac{n_1}{\sqrt{1-n_1^2}}\;.$$

It should be noted that in the case $n_1=0, n_2=0, n_3=1$, the parameter ν equals to zero, and eq. (37) reduces to (16).

Now, consider more general case (compare with (27))

$$\mathbf{n}=(n_1,n_2,n_3)\;,\qquad \mathbf{e}=(1,0,0),\qquad \mathbf{x}_0=(L,0,0)\;; \tag{38}$$

Now, instead of (29) we have

$$\text{sh}\ \beta\sqrt{x^2+y^2+z^2} + x\ \text{ch}\ \beta = \lambda n_1 + L\ , \qquad y = \lambda n_2, \qquad z = \lambda n_3\ . \tag{39}$$

Further, one should act as above: in the plane (y,z) introduce the rotated system

$$z = Z\ \cos\phi = Z\ \frac{n_3}{\sqrt{n_3^2+n_2^2}}, \qquad y = Z\ \sin\phi = Z\ \frac{n_2}{\sqrt{n_3^2+n_2^2}}$$

$$\lambda = \frac{y}{n_2} = \frac{z}{n_3} = \frac{Z}{\sqrt{n_3^2+n_2^2}}\ , \qquad \nu = \frac{n_1}{\sqrt{n_3^2+n_2^2}} = \frac{n_1}{\sqrt{1-n_1^2}}\ ,$$

and from (39) it follows

$$\text{sh}\ \beta\sqrt{x^2+Z^2} = Z\nu - x\ \text{ch}\ \beta + L,\ . \tag{40}$$

Further, instead of (33) we get

$$A = \nu^2 - \text{sh}^2\beta\ , \qquad B = \nu\text{ch}\beta\ ,$$
$$AZ^2 - B\ 2Zx + x^2 + (L^2 - 2xL\text{ch}\beta + 2Z\nu L) = 0\ . \tag{41}$$

Now, let us use the same rotation method:

$$x = \cos\phi X'' + \sin\phi Z'', \qquad Z = -\sin\phi X'' + \cos\phi Z''\ ,$$

Eq. (41) gives

$$(A\ \sin^2\phi + \cos^2\phi + B\ \sin 2\phi)(X'')^2 + (A\cos^2\phi + \sin^2\phi - B\sin 2\phi)\ (Z'')^2$$
$$-[(A-1)\sin 2\phi + 2B\cos 2\phi]X''Z'' +$$
$$+[L^2 - 2(\cos\phi X'' + \sin\phi Z'')\ L\text{ch}\beta + 2(-\sin\phi X'' + \cos\phi Z'')\ \nu L] = 0\ , \tag{42}$$

The angle we need is already known:

$$\tan\phi = \frac{\nu}{\text{ch}\beta}\ , \qquad \cos^2\phi = \frac{\text{ch}^2\beta}{\nu^2+\text{ch}^2\beta}\ , \qquad \sin^2\beta = \frac{\nu^2}{\nu^2+\text{ch}^2\beta}\ .$$

From (41) we get (see (37))

$$(\nu^2+1)(X'')^2 - \text{sh}^2\beta(Z'')^2 +$$
$$+L^2 - 2X''L\ (\text{ch}\beta\cos\phi + \nu\sin\phi) - 2Z''L\ (\text{ch}\beta\sin\phi - \nu\cos\phi) = 0\ . \tag{43}$$

Taking in mind identities

$$\text{ch}\beta\cos\phi + \nu\sin\phi = \frac{\text{ch}^2\beta + \nu^2}{\sqrt{\nu^2+\text{ch}^2\beta}} = \sqrt{\nu^2+\text{ch}^2\beta}\ ,$$

$$\text{ch}\beta\sin\phi - \nu\cos\phi = \frac{\text{ch}\beta\ \ \nu - \nu\ \ \text{ch}\beta}{\sqrt{\nu^2+\text{ch}^2\beta}} = 0\ ,$$

the previous equation reads as

$$(X'')^2 - \frac{\text{sh}^2\beta}{(\nu^2+1)}(Z'')^2 + \frac{L^2}{(\nu^2+1)} - 2X''L\ \frac{\sqrt{\nu^2+1+\text{sh}^2\beta}}{(\nu^2+1)} = 0\ .$$

It remains to perform an elementary shift in X'' variable:

$$\left[X'' - L\,\frac{\sqrt{\nu^2+1+\mathrm{sh}^2\beta}}{(\nu^2+1)}\right]^2 - \frac{\mathrm{sh}^2\beta}{(\nu^2+1)}(Z'')^2 = -\frac{L^2}{(\nu^2+1)} + L^2\,\frac{\nu^2+1+\mathrm{sh}^2\beta}{(\nu^2+1)^2}$$

so that we arrive at the equation determining a hyperbola:

$$\left[X'' - L\,\frac{\sqrt{\nu^2+1+\mathrm{sh}^2\beta}}{(\nu^2+1)}\right]^2 - \frac{\mathrm{sh}^2\beta}{(\nu^2+1)}(Z'')^2 = \frac{L^2\mathrm{sh}^2\beta}{(\nu^2+1)^2} \qquad (44)$$

In special case, when $n_1 = 0, n_2 = 0, n_3 = 1$, the parameter ν becomes equal to zero, and (44) is reduced to (15):

$$(x - L\mathrm{h}\beta)^2 - \mathrm{sh}^2\beta\; z^2 = L^2\mathrm{sh}^2\beta \qquad (45)$$

Let us consider a more general case: let it be

$$\mathbf{n} = (n_1, n_2, n_3)\;, \qquad \mathbf{e} = (1,0,0), \qquad \mathbf{x}_0 = (x_0 = L, y_0, z_0)\;; \qquad (46)$$

at this we have (compare with (39))

$$\mathrm{sh}\;\beta\sqrt{x^2+y^2+z^2} + x\;\mathrm{ch}\;\beta = \lambda n_1 + L\;,$$
$$y = \lambda n_2 + y_0, \qquad z = \lambda n_3 + z_0\;. \qquad (47)$$

The whole analysis of previous case is applied in full, the only difference arises – first one should perform a special shift in the plane (y, z)

$$y - y_0 = \lambda n_2, \qquad z - z_0 = \lambda n_3\;,$$

and then repeat the above calculation.

In conclusion, let us formulate general method to modify geometrical form of any rigid curve (not only a direct line) while changing the reference frame. Let in the rest reference frame K' a curved line is determined by two relations:

$$K' : \qquad \varphi_1(\mathbf{x}) = 0\;, \qquad \varphi_2(\mathbf{x}) = 0\;. \qquad (48)$$

Evidently, the above receipt can apply in this general case too – it gives two equations

$$K : \qquad \varphi_1\{\mathbf{x} + \mathbf{e}\;[\;\mathbf{sh}\;\beta\;\sqrt{\mathbf{x^2}} + (\mathbf{ch}\;\beta - \mathbf{1})\;\mathbf{ex}\;]\} = 0\;,$$
$$\varphi_2\{\mathbf{x} + \mathbf{e}\;[\;\mathbf{sh}\;\beta\;\sqrt{\mathbf{x^2}} + (\mathbf{ch}\;\beta - \mathbf{1})\;\mathbf{ex}\;]\} = 0\;. \qquad (49)$$

They determine geometrical form of the curved line in the moving reference frame.

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