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Gluon dominance model and e^+e^- annihilation

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Gluon dominance model (GDM) studies multiparticle production in lepton and hadron processes. It is based on the essentials of QCD and the main experimental phenomena in these processes. The model describes multiplicity distributions and their moments rather well. An active role of gluons was revealed in multiparticle production and the fragmentation mechanism of hadronization in e^+e^- annihilation has been confirmed.

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1. Introduction

At present the development of the contemporary theory of nuclear matter is continued. This is due to the latest experiments on Quark-Gluon Plasma (QGP) search [1].

From 80-es the QGP conception has undergone considerable changes. Novel experimental results were obtained in CERN (SPS) [2] and BNL (RHIC) [3]. To explain extraordinary phenomena in these processes, different approaches are used on QCD [4], Color Glass Condensate [5], hydrodynamical [6], statistical [7] and other models [8].

There is no unified theory or any acceptable model which could explain all the results obtained at high energies. Presumably we should find the answer on this difficult problem by studying of multiparticle production (MP) in hadron and nucleus interactions at lower energies. Till the present moment "soft" hadronic events named by P. Carruthers [9] "ugly duckling" of high energy physics, have not been understood completely. He also stressed, that relativistic heavy ion (RHI) investigations would become frontiers of the high energy physics.

The gluon dominance model (GDM) [10] realizes a unified approach to study the multiplicity behavior in different interactions by using QCD essentials and a phenomenological approach to hadronization. We have shown that this study agrees with the experimental data from a wide energy region, and perhaps, can be helpful to analyze new phenomena. The calculations of the multiplicity distribution (MD) in e^+e^- annihilation are in section 2. The descriptions of moment oscillations and second correlative moments are shown in sections 3 and 4. Summary contains the main results.

2. Multiplicity distributions

The e^+e^- annihilation is one of the most suitable processes to study MP [11]. According to the theory of strong interactions QCD it can be realized through production of γ or Z^0–boson into two quarks:

$$e^+e^- \to (Z^0/\gamma) \to q\bar{q}. \tag{1}$$

The e^+e^-–reaction is simple for analysis, since the produced state is pure $q\bar{q}$. It is usually difficult to determine the quark species on event-by-event basis, therefore the average over

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the quark type is generally applied. Perturbative QCD can describe the process of fission partons (quarks and gluons) at high energy, because strong coupling α_s is small at high energy [12]. This stage can be called as "cascade". By the end of the fission process partons have small virtuality and must transform into hadrons which we observe. At this stage we could not apply perturbative QCD. Therefore phenomenological models are used to describe the hadronization process (transformation of quarks and gluons into hadrons). They are built on the experimental data obtained from the study of low–$\mathbf{p_T}$ hadron collisions. It is usually considered that production of hadrons from partons is a universal process.

Parton spectra of quark and gluon jets in QCD were studied by K. Konishi, A. Ukawa and G. Veneciano. Working at the leading logarithm approximation and avoiding IR divergences by considering finite x, the probabilistic nature of the problem has been established [13].

Studying MP at high energy we took the idea of A. Giovannini ([14], [15]) to describe quark-gluon jets as markovian branching processes. He proposed to interpret the natural QCD evolution parameter $\mathbf{Y}$

$$\mathbf{Y} = \frac{1}{2\pi b}\ln[1+\alpha_s b \ln(\frac{Q^2}{\mu^2})], \tag{2}$$

where $2\pi b = \frac{1}{6}(11N_C - 2N_f)$ for a theory with N_C colours and N_f flavours, as the thickness of the jet.

Three elementary processes contribute into QCD quark-gluon jets: (1) gluon fission; (2) quark bremsstrahlung; (3) quark pair production. According to [14] $A\Delta Y$ is the probability that gluon will convert into two gluons in the infinitesimal interval ΔY, $\widetilde{A}\Delta Y$ is the probability that quark will radiate a gluon, and $B\Delta Y$ is the probability that a quark-antiquark pair will be produced from a gluon. $A, \widetilde{A}, B$ are assumed to be $\mathbf{Y}$-independent constants and each individual parton acts independently of others, always with the same infinitesimal probability.

A.Giovannini constructed a system of differential equations and obtained explicit solutions for the parton jets in the case $B = 0$ (the third process is absent). MD of the quark jet are given: [14]

$$P_0(Y) = e^{-\widetilde{A}Y},$$

$$P_m(Y) = \frac{\mu(\mu+1)\dots(\mu+m-1)}{m!}e^{-\widetilde{A}Y}(1-e^{-AY})^m, \tag{3}$$

where $\mu = \widetilde{A}/A$, $\overline{m} = \mu(e^{AY}-1)$ is the average gluon multiplicity. These MD are known as negative binomial distributions (NBD). Their generating function (GF) is given as follows:

$$Q^q(z,Y) = \sum_{m=0}^{\infty} z^m P_m(Y) = [1+\overline{m}/\mu(1-z)]^{-\mu}. \tag{4}$$

In 90-es we called a scheme which joint the quark-gluon cascade and hadronization as a Two Stage Model (TSM) [10]. The next study of pp interactions revealed the active role of gluons at MP and led to the change of TSM to the Gluon Dominance Model (GDM) [16], [17]. In TSM we took (3) to describe a cascade stage and added a super narrow binomial distribution (BD) for the hadronization stage. This choice was based on the experimental data analysis in e^+e^--annihilation lower than 9 GeV. Second correlation moments were negative at these energies. The choice of these distributions was the only one possible to describe the experiment.

We supposed that the hypothesis of soft decoloration was right. That is why we united the both stages by means of convolution. MD in this case are written as

$$P_n(Y) = \sum_m P_m^P(Y)P_n^H(m), \tag{5}$$

where $P_m^P(Y)$ is MD for partons (3), $P_n^H(m)$ - MD for hadrons produced from m partons at the hadronization stage. Further we chose a center of masses energy $\sqrt{s}$ instead of variable $\mathbf{Y}$.

In accordance with TSM the stage of hard fission of partons is described with NBD for the quark jet

$$P_m^P(s) = \frac{k_p(k_p+1)\dots(k_p+m-1)}{m!}\left(\frac{\overline{m}}{\overline{m}+k_p}\right)^m\left(\frac{k_p}{k_p+\overline{m}}\right)^{k_p}, \tag{6}$$

where $k_p = \widetilde{A}/A$, $\overline{m} = \sum_m mP_m^P$. Two quarks are fragmenting to partons independently of each other. MD P_m^P and GF for these MD $Q^P(s,z)$ are equal to:

$$P_m^P = \frac{1}{m!}\frac{\partial^m}{\partial z^m}\,Q^P(s,z)\big|_{z=0}\,, \tag{7}$$

$$Q_m^P(s,z) = \left[1+\frac{\overline{m}}{k_p}(1-z)\right]^{-k_p}. \tag{8}$$

MD of hadrons formed from one parton are described in the following form [10]

$$P_n^H = \binom{N_p}{n}\left(\frac{\overline{n}_p^h}{N_p}\right)^n\left(1-\frac{\overline{n}_p^h}{N_p}\right)^{N_p-n}, \tag{9}$$

with generating function

$$Q_p^H = \left[1+\frac{\overline{n}_p^h}{N_p}(z-1)\right]^{N_p}, \tag{10}$$

where $\overline{n}_p^h$ and N_p $(p=q,g)$ mean the average multiplicity and a maximum possible number of secondary hadrons formed from parton at the hadronization stage.

We should mention the object "gluer" which was introduced by Yu.L. Dokschitzer and others [18] and compare to "our" gluon at the second stage. Since this region is beyond consideration of the perturbation theory, gluons radiate strongly, they are hardly treated and participate in the hadronization immediately after being formed. They named this object "gluer", stressing that it is the privilege of gluers but not gluons to glue. Also we would like to emphasize the exclusive role of the PINP-group in the construction of the probabilistic scheme for the parton and hadron jets, basing on the hypothesis of the local parton-hadron duality (LPHD), the evolution of the quantitative description of the modified leading logarithmic approximation (MLLA).

Our study is aimed at investigating the soft region where an active gluon (a gluer) turns into real hadrons.

Thus, MD of hadrons in e^+e^- annihilation are determined by the convolution of two stages (cascade and hadronization):

$$P_n(s) = \sum_{m=o}^{\infty} P_m^P(s)\frac{1}{n!}\frac{\partial^n}{\partial z^n}(Q^H)^{2+m}\Bigg|_{z=0}, \tag{11}$$

Table 1. Parameters of TSM.

$\sqrt{s}$ GeV	$\overline{m}$	k_p	N	$\overline{n}^h$	α	χ^2
14	0.08	$2 \cdot 10^8$	27.7	2.87	.970	2.75
22	3.01	4.91	20.2	4.34	.200	1.29
34.8	6.58	6.96	12.5	4.10	.195	240
43.6	10.30	48.30	5.16	2.31	.444	5.36
50	7.48	1.30	24.6	6.14	.100	1.97
52	11.50	1.00	24.8	6.16	.104	2.51
55	8.60	$6 \cdot 10^4$	17.0	4.00	.260	124
56	9.81	8.23	6.51	3.73	.273	1.29
57	11.30	14.40	4.00	2.76	.385	1.95
60	8.92	9.00	9.31	4.20	.254	2.97
60.8	9.52	6.68	7.00	4.12	.246	2.02
61.4	10.40	1.00	21.1	6.38	.108	1.76
91.4	10.90	7.86	11.2	4.80	.226	1.16
133	12.00	2.99	17.1	6.50	.140	4.20
161	3.47	20.20	10.4	6.23	.550	4.45
172	20.10	9.11	9.17	4.34	.195	6.86
183	13.20	1.48	54.6	8.90	.086	2.56
189	15.10	6.90	11.6	5.15	.215	2.37

where $2 + m$ is the total number of partons (two quarks and m gluons).

Further we fulfil the following assumption for the second stage: $\overline{n}_q^h/N_q \approx \overline{n}_g^h/N_g$, considering the probabilities of hadron formation from quark and gluon to be equal. We introduce parameter $\alpha = N_g/N_q$ to distinguish hadron jets produced from quark and gluon. After some of the simplifications in the designations ($N = N_q$, $\overline{n}^h = \overline{n}_q^h$) we get

$$Q_{q(g)}^H = \left(1 + \frac{\overline{n}^h}{N}(z-1)\right)^{(\alpha)N}.$$

Introducing expressions (6), (10) in (11) and differentiating on z, we obtain MD of hadrons in the process of e^+e^- annihilation in TSM:

$$P_n(s) = \sum_{m=0}^{M_g} P_m^P \binom{(2+\alpha m)N}{n} \left(\frac{\overline{n}^h}{N}\right)^n \left(1 - \frac{\overline{n}^h}{N}\right)^{(2+\alpha m)N-n}. \tag{12}$$

To compare the experimental data, the number of gluons in the above sum was restricted by M_g - the maximal number of possible gluons produced at the first stage. Summing up is limited by M_g equal to $20 - 22$ for energies up to 61.4 GeV and $33 - 41$ at higher, because further increase of them does not change χ^2. The results of the comparison of the expression (12) with the experimental data [19] are shown in Table 1 (errors about 5%) and in Fig. 1. We can see that MD in TSM (solid curve) describe the experimental data from 14 to 189 GeV [20] very well.

We can show the following MP picture at the first cascade stage. The average gluon multiplicity $\overline{m}$ has a tendency to rise, but lower than the logarithmic one [20]. In [14] $\overline{m}(s)$ grows also, k_p, which is equal to the ratio $2\tilde{A}/A$, approaches to 1. In our case these values remain of the order of 10. The physical sense of k_p as the temperature T was given in [21]: $T \sim k_p^{-1}$,

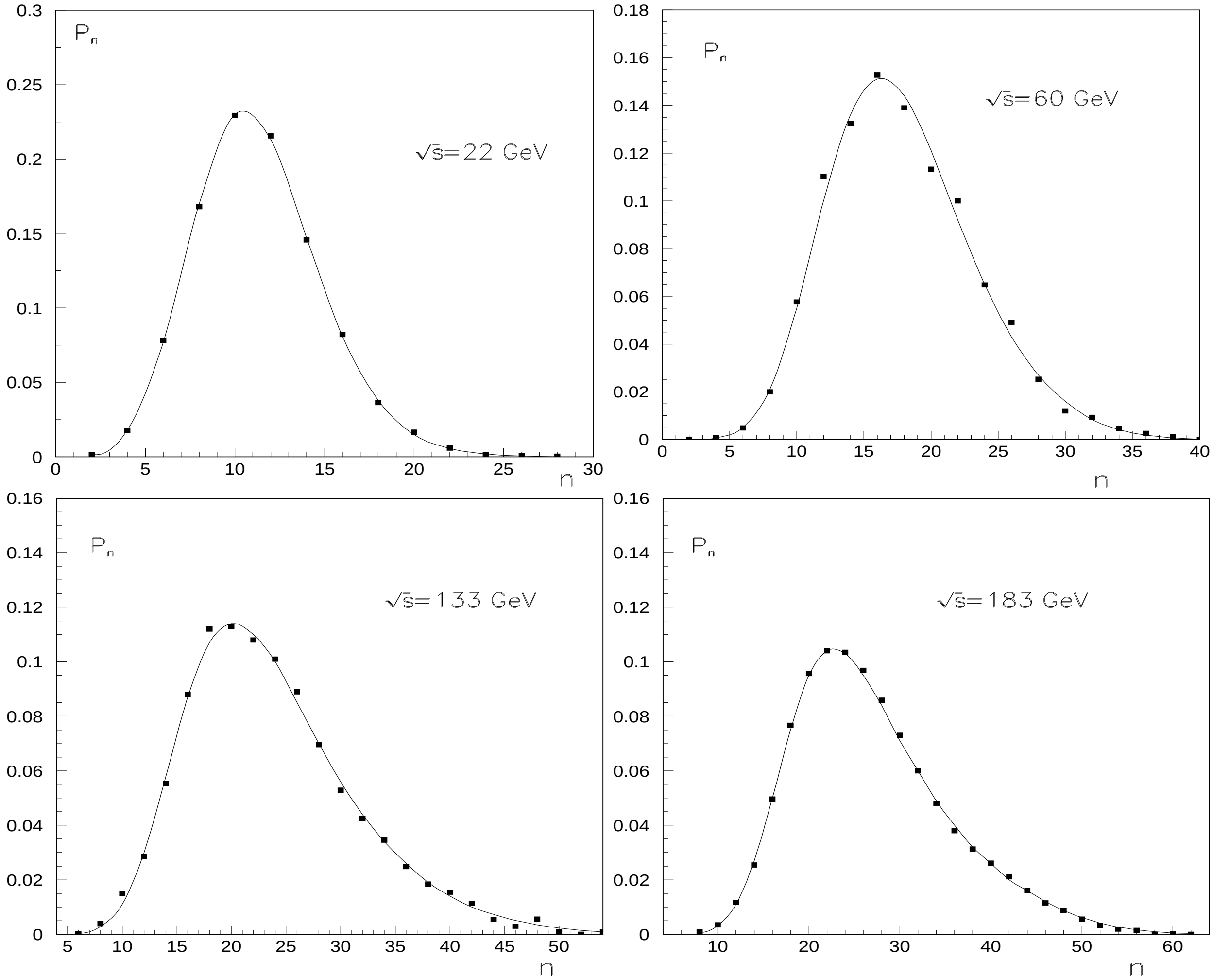

FIG. 1. MD (12) at 22 (TASSO Coll.), 56 (AMY Coll.), 91 (DELPHY Coll.) and 183 GeV (OPAL Coll.).

$k_p^{-1} \sim T_0 + 1/cE$, where T_0 is the temperature of the system before the interaction, where c is thermal capacity, E - the energy for new particles production. We can estimate the parton system temperature at different energies. It has a tendency to increase with the energy.

A curious picture of hadronization has been found according to the behaviour of its three parameters. The first parameter N_q determines the maximum number of secondary hadrons formed from a quark while its passing through this stage. We suppose that the probabilities of gluon fission, quark bremsstrahlung and quark pair production become comparable between themselves. Then the quark pairs production results to the hadron production. N_q takes different values from 4 to 55, so, we cannot conclude that the stable energy behavior is observed.

The second parameter $\overline{n}_q^h$ is the number of hadrons formed from quark at this stage. We can observe the weak tendency to rise. The parameter α was introduced to compare quark and gluon jets. It is almost constant and equal to 0.2 with some deviations. If we know α, then we can determine gluon parameters $N_g = \alpha N_q$ and $\overline{n}_g^h = \alpha \overline{n}_q^h$. It is a surprise that they remain almost constant: $N_g \sim 3$ and $\overline{n}_g^h \sim 1$ (Fig. 2). From this result we can conclude that the mechanism of the gluon hadronization is universal and fragmentational [22] in e^+e^- annihilation processes. The fact that $\alpha < 1$ proves that hadronization of the gluon jet is carried out "softer" than the quark jet.

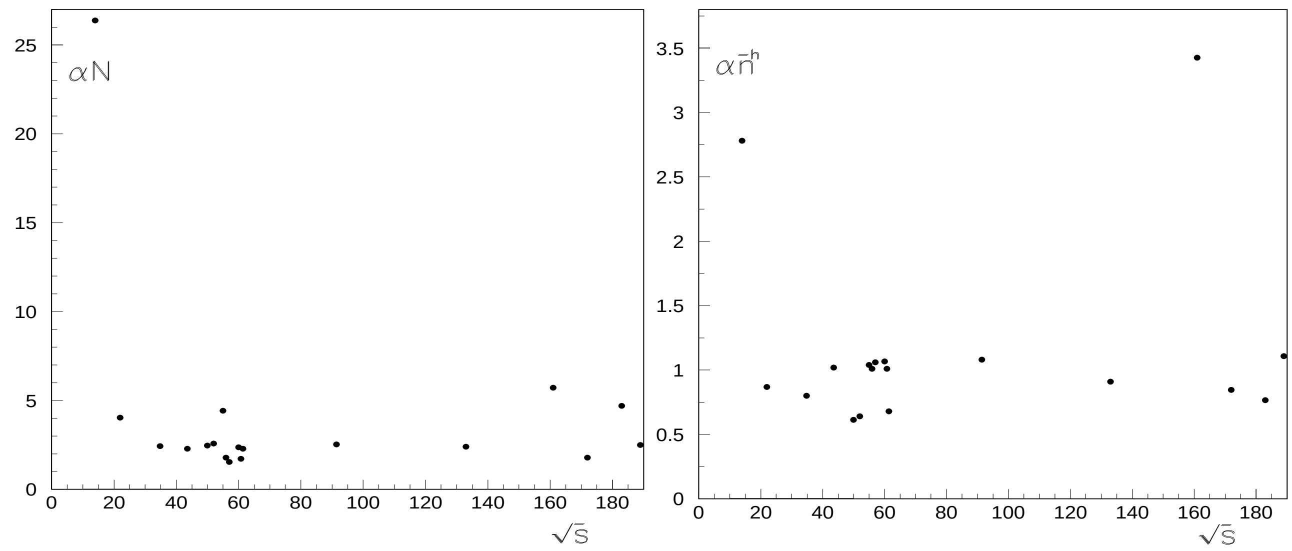

FIG. 2: The gluon hadronization parameters N_g (left) and $\overline{n}_g^h$ (right).

3. Oscillations of the moments

It is known [23] that the ratio of factorial cumulative moments over factorial moments changes the sign as a function of the order. We use MD formed in TSM to explain this phenomenon. The factorial moments can be obtained from MD P_n by means of the following equation:

$$F_q = \sum_{n=q}^{\infty} n(n-1)\dots(n-q+1)P_n, \qquad (13)$$

and factorial cumulative moments are found from expression

$$K_q = F_q - \sum_{i=1}^{q-1} C_{q-i}^{i} K_{q-i} F_i. \qquad (14)$$

The ratio of these values is

$$H_q = K_q/F_q. \qquad (15)$$

The generating function for MD of hadrons (12) in e^+e^- annihilation G(z) is the following convolution:

$$G(z) = \sum_{m=0} P_m^g [Q_g^H(z)]^m Q_q^2(z) = Q^g(Q_g^H(z))^m Q_q^2(z). \qquad (16)$$

We calculate F_q and K_q in TSM by using (16)

$$F_q = \frac{1}{\overline{n}^q(s)} \left.\frac{\partial^q G}{\partial z^q}\right|_{z=1}, \quad K_q = \frac{1}{\overline{n}^q(s)} \left.\frac{\partial^q \ln G}{\partial z^q}\right|_{z=1}. \qquad (17)$$

After taking a logarithm the expression(16) for G(z)

$$\ln G(s,z) = -k_p \ln[1 + \frac{\overline{m}}{k_p}(1 - Q_g^H)] + 2\ln Q_q^H$$

and its further expansion to series in power on Q_g^H, will be as follows:

$$\ln G(s,z) = k_p \sum_{m=1} \left(\frac{\overline{m}}{\overline{m}+k_p}\right)^m \frac{Q_g^m}{m} + 2\ln Q_q^H. \qquad (18)$$

Inserting Q_g and Q_q^H into (18)

$$\ln G(s,z) = k_p \sum_{m=0} \left(\frac{\overline{m}}{\overline{m}+k_p}\right)^m \frac{1}{m}\left[1+\frac{\overline{n}^h}{N}(z-1)\right]^{\alpha m N} + 2N\ln[1+\frac{\overline{n}^h}{N}(z-1)],$$

and using (17) we obtain

$$K_q = \left(k_p \sum_{m=1} \alpha(\alpha m - \frac{1}{N})\ldots(\alpha m - \frac{q-1}{N})\cdot\left(\frac{\overline{m}}{\overline{m}+k_p}\right)^m -2(-1)^q\frac{(q-1)!}{N^{q-1}}\right)\left(\frac{\overline{n}^h}{\overline{n}(s)}\right)^q, \quad (19)$$

where $\overline{n}(s)$ is the average multiplicity hadrons in process (1). It is possible to find F_q using (17)

$$F_q = \sum_{m=0}(2+\alpha m)(2+\alpha m-\frac{1}{N})\ldots(2+\alpha m-\frac{q-1}{N})P_m\left(\frac{\overline{n}^h}{\overline{n}(s)}\right)^q, \quad (20)$$

where P_m is equal to (6). The sought-for expression for H_q will be the following:

$$\begin{aligned} H_q \;=\; & \left[\sum_{m=1} k_p\alpha(\alpha m-\frac{1}{N})\ldots(\alpha m-\frac{q-1}{N})\left(\frac{\overline{m}}{\overline{m}+k_p}\right)^m - 2(-1)^q\frac{(q-1)!}{N^{q-1}}\right] \\ & \Big/ \left[\sum_{m=0}(2+\alpha m)(2+\alpha m-\frac{1}{N})\ldots(2+\alpha m-\frac{q-1}{N})P_m\right]. \end{aligned} \quad (21)$$

The comparison with the experimental data [23] has shown that (21) describes the ratio of factorial moments (Fig. 3). The function minimum is seen at q=5.

In the region lower Z^0, H_q oscillates in the sign only with the period equal to 2 and changes the sign with parity q. Beyond this region the oscillation period increases till 4 and higher. It can be explained by the influence of the cascade development and hadronization. Values K_q (as well H_q) may change the sign only due to the second summand in (19). Taking into account hadronization and the more developed cascade of partons leads to the increase of the value of the oscillation period. Significant oscillations can be explained by the convolution of the wide parton cascade and super narrow BD for hadronization at the second stage.

It is necessary to add that analysis of MP in framework of the another picture based on dissipating energy of participants [24] describes the similarity of the bulk observable as the mean multiplicity in hadron (nucleus) and e^+e^- interactions.

4. Second correlative moment f_2

By definition the second correlative moments f_2 are

$$f_2 = \overline{n(n-1)} - \overline{n}^2.$$

They could be calculated with GF

$$f_2 = \left.\frac{\partial^2 G(s,z)}{\partial z^2}\right|_{z=1} - \left(\left.\frac{\partial G(s,z)}{\partial z}\right|_{z=1}\right)^2. \quad (22)$$

The immediate use of GF G(s,z) (16) leads to

$$f_2 = \left(\alpha^2\frac{\overline{m}^2}{k_p} + \alpha^2\overline{m} - \frac{2+\alpha\overline{m}}{N}\right)(\overline{n}^h)^2. \quad (23)$$

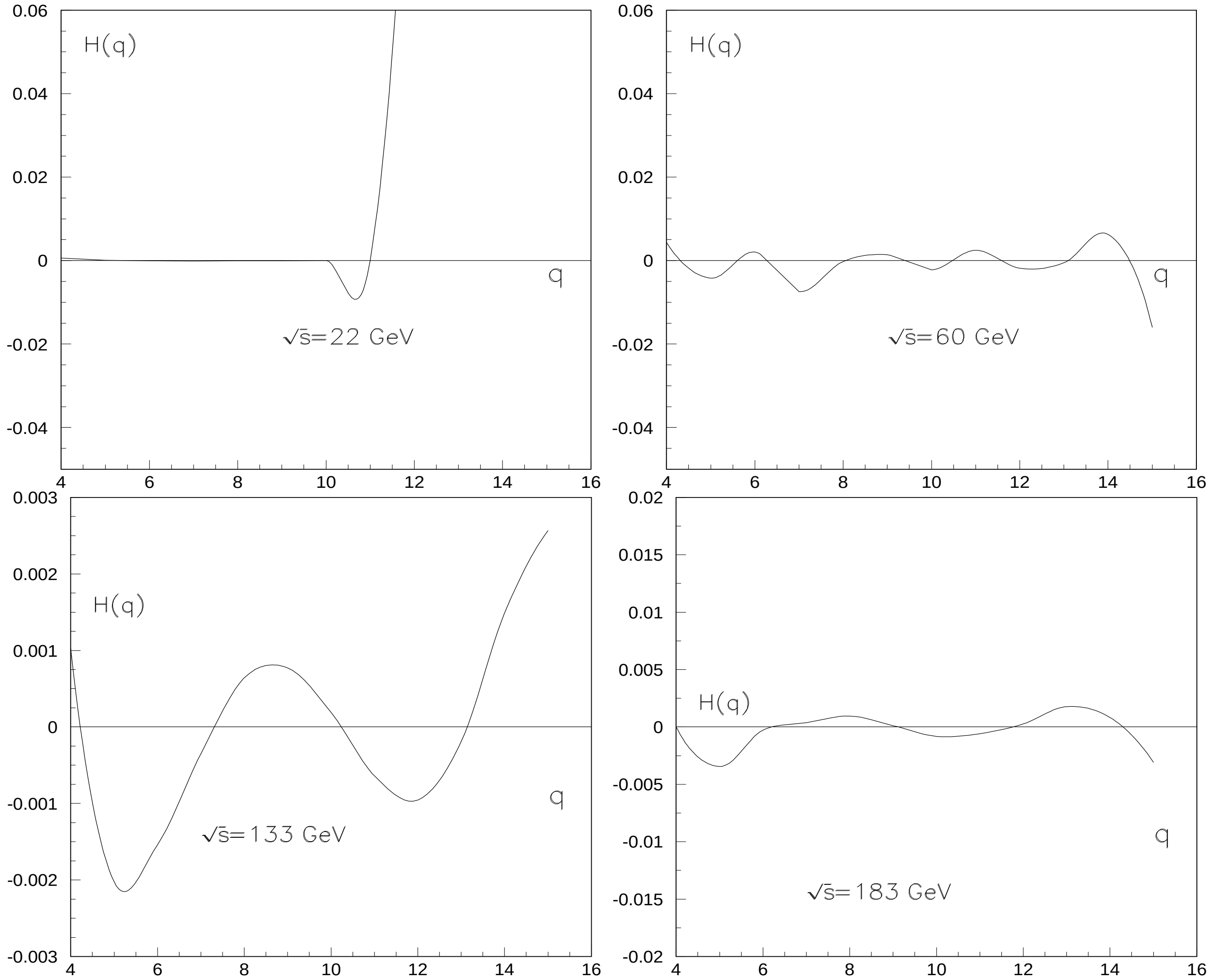

FIG. 3: H_q (21) at 22, 60, 133 and 183 GeV.

The energy dependence of f_2 agrees with the experimental results [19]. At low energy the mean number of gluons $\overline{m}$ is very small (an undeveloped parton cascade), parameter $k_p >> 1$ and the second correlative moment has a negative value (the hadronization predominance). At high energy f_2 becomes positive, because it is defined as the first term in (23) since $\overline{m} > 1$ and $k_p \sim 10$. So, we can see the change of the correlative moment sign with energy increase from negative values to the positive ones.

5. Summary

The proposed approach to the MP study in e^+e^- annihilation allows me to investigate the parton cascade and hadronization, to describe MD and their moments. Also the permanence of the hadronization parameters for gluons was established that is the evidence of the fragmentation character of hadronization in e^+e^- annihilation.

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References

[1] J. Adams et al. STAR Collaboration. Experimental and theoretical challenges in the search for the quarkgluon plasma: The STAR Collaboration's critical assessment of the evidence from RHIC collisions. Nucl.Phys. **A757**, 102-183 (2005).

[2] B. Muller. Heavy ions - Prospects at LHC. Czech.J.Phys. **55**, B312-B323 (2005).

[3] P. Jacob and Xin-Nian Wang. Matter in extrimis: ultrarelativistic nuclear collisions at RHIC. Prog.Par.Nucl.Phys. **54**, 443-534 (2005).

[4] Yu.L. Dokshitzer. QCD phenomenology. International CERN-Dubna school, Pylos (2002);
V.A. Hoze, W. Ochs and J.Wosiek. Analytical QCD and multiparticle production. [hep-ph/0009298].

[5] L. D. McLerran The Quark Gluon Plasma and the Color Glass Condensate: 4 Lectures. [hep-ph/0311028];
D. Harzeev and M. Nardi. Hadron production in nuclear collisions at RHIC and high density QCD. Phys.Lett.**B507**,121-128 (2001);
D.Kharzeev and E.Levin. Manifestations of high density QCD in the first RHIC data. Phys.Lett.**B523**, 79-87 (2001).

[6] P. F. Kolb and U. Heinz. Hydrodinamic description of ultrarelyativistic heavy ions collisions. Quark gluon plasma 3. World scientific, Singapore. [nucl-th/0305084].

[7] P. Braun-Munzinger, K. Redlich and J. Stashel. Particle production in heavy ion collisions. Quark gluon plasma 3. World scientific, Singapore.[nucl-th/0304013].
R. Hagedorn. Statistical thermodynamics of strong interactions at high energies. Nuovo Cim.Suppl.**3**, 147-186 (1965).

[8] M. Gyulassy et al. Deflagrations and detonations as a mechanism of hadron bubble growth in supercooled quark gluon plasma. Nucl.Phys.**B237**,477-501 (1984);
L.Csernai and Joseph I. Kapusta. Entropy and cluster production in nuclear collisions. Phys.Rep. **131**,223-318 (1986).

[9] P. Carruthers. The desertron as gluon factory. Preprint LA-UR-84-1084 (1984).

[10] V.I. Kuvshinov, E.S. Kokoulina. Charged hadron correlations in quark and gluon jets. Acta Phys.Polon., **B13**, 553-558 (1982);
E.S. Kokoulina, V.I. Kuvshinov V.I. Charged hadron correlations in quark and gluon jets. Proceedings. Multiquark Interactions and QCD. JINR, Dubna, $\mathbf{D1}, \mathbf{2-81}$, 299-304 (1981).

[11] O.Biebel. Experimental tests of the strong interaction and its energy dependence in electron positron annihilation. Phys.Rep., **340**, 165-289 (2001).

[12] H.P. Politzer. Asymptotic freedom: an approach to strong interactions. Phys.Rep. **14** 129-180 (1974).

[13] K. Konishi, A. Ukawa, G. Veneciano. Jet calculus: a simple algorithm for resolving QCD jets. Nucl.Phys. **B157**, 45-107 (1979).

[14] A. Giovannini. QCD jets as Markov branching processes. Nucl.Phys. **B161**, 429-448 (1979).

[15] A. Giovannini and R. Ugocioni. Clan structure analysis and QCD parton showers in multiparticle dynamics: an intriguing dialog between theory and experiment. Int.J.Mod.Phys. **A20** 3897-4000 (2005).

[16] E.S. Kokoulina and V.A.Nikitin. Study of multiparticle production by gluon dominance model. Proc. 17*th* ISHEPP, 319-326 (2005).JINR, Dubna;
P.F. Ermolov et al. Study of multiparticle production by gluon dominance model. Part II. Proc. 17*th* ISHEPP, 327-336 (2005), JINR,Dubna.

[17] E.S. Kokoulina. Gluon dominance model. AIP Conf.Proc. **828**, 81-86 (2006);
E.S. Kokoulina. Gluon dominance model and multiparticle production. 8th Int. Workshop on RNP: From Hundreds MeV to TeV, 272-277 (2006).

[18] Yu.L. Dokschitzer, V.A. Khoze, A.H. Muller and S.I. Troyan, Basics of Perturbarive QCD, Editions Fronieres, 1991.

[19] W. Braunschweig et al. TASSO Collaboration. Charged multiplicity distributions and correleations in e^+e^- annihilation at PETRA energies. Z.Physik **C45**, 193-244 (1989);
H.W. Zheng et al. Charged hadron multiplicities in e^+e^- annihilations at $\sqrt{s} = 50 - 61.4$GeV. AMY Collaboration. Phys.Rev. **D42**, 737-747 (1990);
P. Abreu et al. DELPHY Collaboration. Charged particle multiplicity distributions in restricted rapidity intervals in Z0 hadronic decays. Z.Phys. **C52**, 271-281 (1991);
P.D. Acton et. al. OPAL Collaboration. A study of charged particle multiplicities in hadronic decays of the Z0. Z.Physik. **C53**, 539-554 (1992);
G. Alexander et al. OPAL Collaboration. QCD studies with e+ e- annihilation data at 130-GeV and 136-GeV. Z.Physik. **C72**, 191-206 (1996);
K. Acketstaff et al. OPAL Collaboration. QCD studies with e+ e- annihilation data at 161-GeV. Z.Physik. **C75**, 193-207 (1997);
G. Abbiendi et al. OPAL Collaboration. QCD studies with e^+e^- annihilation data at $172 - 189$ GeV. Eur.Phys.J. **C16** 185-210 (2000).

[20] E.S. Kokoulina. The Description of multiplicity distributions in high-energy e+ e- annihilation within the framework of two stage model. 6th International School - Seminar On Actual Problems Of High-Energy Physics , Gomel, (2001);
E.S. Kokoulina. Multiplicity distribution studies of e+ e- annihilation at 50-61.4-GeV and 172-189-GeV by two stage model hadronization. XI Annual Seminar NPCS, Minsk, Belarus, (2002).

[21] E.S. Kokoulina, V.I. Kuvshinov. Multiplicity distribution behavior analys in hadron interactions at high energies. Vesti AN BSSR. Ser.Phys.-Math.Sc. **3**, 49-54 (1988).

[22] B.Müller. Hadronic signals of deconfinement at RHIC. Contribution to RBRC Scientific Articles Proceedings Series "New Discoveries at RHIC" (2004),[nucl-th/0404015].

[23] I.M. Dremin, R.G. Hwa. Quark and gluon jets in QCD: Factorial and cumulant moments. Phys.Rev. **D49**, 5805-5811 (1994);
K. Abe et al. SLD Collaboration. Factorial and cumulant moments in $e^+e^- \to$ hadrons at the Z^0 resonance. Phys. Lett. **B371**, 149-156 (1996).

[24] E.K.G. Sarkisyan, A.S. Sakharov. Multihadron production features in different reactions. AIP Conf.Proc. **828**, 35-41 (2006).