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Distribution amplitudes of light mesons and photon in the instanton model*

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The leading- and higher-twist distribution amplitudes of pion, ρ-meson and real and virtual photons are analyzed in the instanton liquid model.

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1. Introduction

Investigations of hard exclusive processes are essential for our understanding of the internal quark-gluon dynamics of hadrons. Theoretically, such studies are based on the assumption of factorization of dynamics at long and short distances. The short-distance physics is well elaborated by perturbative methods of QCD and depends on particular hard subprocesses. The long-distance dynamics is essentially nonperturbative and within the factorization formalism becomes parametrized in terms of hadronic *distribution amplitudes* (DAs). These nonperturbative quantities are universal and are defined as vacuum-to-hadron matrix elements of particular nonlocal light-cone quark or quark-gluon operators. The evolution of DAs at sufficiently large virtuality q^2 is controlled by the renormalization scale dependence of the quark bilinear operators within the QCD perturbation theory. For leading-order DAs this dependence is governed by QCD evolution equations. When the normalization scale goes to infinity the DAs reach an ultraviolet fixed point and are uniquely determined by perturbative QCD. However, the derivation of the DAs themselves at an initial scale μ_0^2 from first principles is a nonperturbative problem and remains a serious challenge.

Here we present the results [1, 2] of study of the pion, ρ-meson and photon DAs in the leading and higher twists at a low-momentum renormalization scale in the gauged non-local chiral quark model [3–5] based on the instanton picture of QCD vacuum.

2. Definitions and notations

The distribution amplitudes of the mesons or the photon are defined via the matrix elements of quark-antiquark bilinear operators taken between the vacuum and the hadronic state $|h(q)\rangle$ of momentum q. It is assumed that the quark and antiquark are separated by the distance $2z$ and the light-like limit $z^2 \to 0$ is taken at a fixed scalar product $q \cdot z$. We use the light-cone expansion of the matrix elements in order to define the DAs (only leading twist terms are presented)

$$\langle 0|\overline{d}(z)\gamma_\mu\gamma_5[z,-z]u(-z)|\pi^+(q)\rangle = i\sqrt{2}f_\pi q_\mu\int_0^1 dx e^{i\xi q\cdot z}\phi_\pi^{\mathrm{A}}(x), \tag{1}$$

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$$\langle 0|\overline{q}(z)\sigma_{\mu\nu}[z,-z]q(-z)|\gamma^\lambda(q)\rangle = ie_q \langle 0|\overline{q}q|0\rangle \chi_{\rm m} \cdot \\ \cdot f^t_{\perp\gamma}(q^2)\left(e^{(\lambda)}_\mu q_\nu - q_\mu e^{(\lambda)}_\nu\right)\int_0^1 dx e^{i\xi q\cdot z}\phi_{\perp\gamma}(x,q^2), \tag{2}$$

$$\langle 0|\overline{q}(z)\gamma_\mu[z,-z]q(-z)|\gamma^\lambda(q)\rangle = e_q f_{3\gamma} f^v_{\|\gamma}(q^2) q_\mu \cdot \\ \cdot \frac{e^{(\lambda)}\cdot z}{q\cdot z}\int_0^1 dx e^{i\xi q\cdot z}\phi_{\|\gamma}(x,q^2), \tag{3}$$

where f_π is the pion decay constant, $\langle 0|\overline{q}q|0\rangle$ is the quark condensate, $\chi_{\rm m}$ is the magnetic susceptibility of the quark condensate, and $f_{3\gamma}$ is related to the first moment of the magnetic susceptibility. The symbol $[-z,z]$ in the matrix elements denotes the path-ordered gauge link (Wilson line) for the gluon fields between the points $-z$ and z. The integration variable x corresponds to the momentum fraction carried by the quark and $\xi = 2x-1$ for the short-hand notation. For a real photon, due to condition $e^{(\lambda)}\cdot z = 0$, the structure corresponding to $\phi_{\|\gamma}$ decouples. The DAs $\phi_{\perp\rho}(x)$ and $\phi_{\|\rho}(x)$ for the ρ-meson state $|\rho^\lambda(q)\rangle$ are defined in analogy with photon case (2) and (3) with mass-shell condition $q^2 = -M_\rho^2$. Note that our definitions of the photon and ρ-meson DAs follow closely the works of Braun, Ball and coauthors [6, 7].

3. Instanton-motivated nonlocal chiral quark model

In the one loop approximation the quark model evaluation of the distribution function $\phi_{h,J}(x)$ of hadron h corresponding to projection J is given schematically as [13]

$$N_{h,J}\phi_{h,J}(x) = -iN_c\int d\tilde{k}\delta(k\cdot n - x)\mathrm{Tr}[\Gamma_J S(k)\Gamma_h S(k-q)], \tag{4}$$

where the quark propagator has the form

$$S(p) = \frac{1}{\widehat{p} - M(p) + i\varepsilon}, \qquad M(p) = M_0 f^2(p^2), \tag{5}$$

with the dynamical quark mass $M(p)$ expressed via the function $f(p)$ defining the nonlocal properties of the QCD vacuum [14]. Γ_h are the vertices defining the hadron state

$$\Gamma_\pi(k,k') = \gamma_5 f(k) f(k'), \quad \Gamma^\mu_\rho(k,k') = \gamma^\perp_\mu f_V(k) f_V(k'), \\ \Gamma^\mu_\gamma(k,k') = \gamma_\mu - (k+k')_\mu M^{(1)}_{k,k'}, \tag{6}$$

and Γ_J is the projection operator corresponding to a definite twist. Here and below, the notation

$$M^{(1)}(k,k') = \frac{M(k) - M(k')}{k^2 - k'^2}$$

is used. The nonlocal functions are chosen in gaussian form

$$f(p) = f_V(p) = \exp\left(-\frac{p^2}{\Lambda^2}\right), \tag{7}$$

with p denoting the Euclidean momentum, resembling the fact that the instanton field is convenient to take in the axial gauge. As the model parameters we take the values fixed in [12]

$$M_0 = 240 \text{ MeV}, \quad \Lambda = 1110 \text{ MeV}. \tag{8}$$

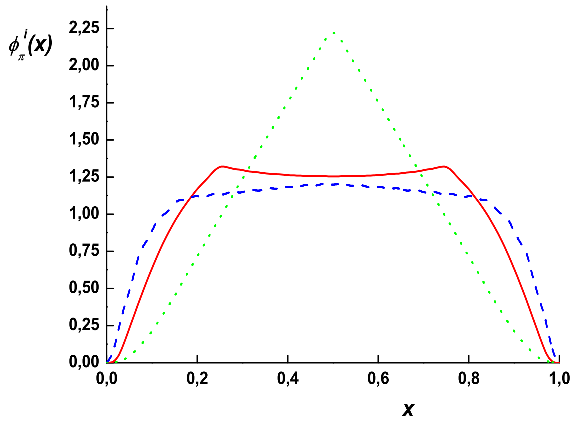

FIG. 1. Pion distribution amplitudes: twist-2 axial-vector (solid line), twist-3 pseudoscalar (short-dashed) and tensor (dotted) projections, given at the quark model scale.

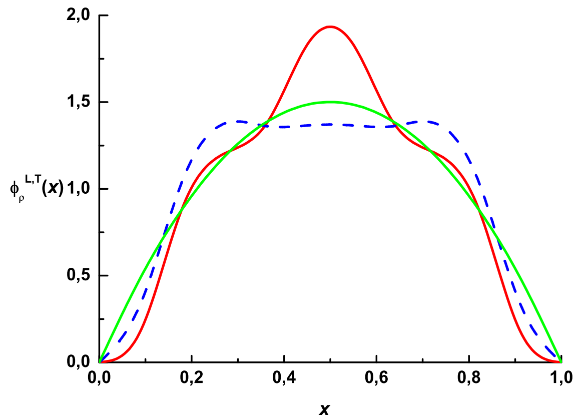

FIG. 2. ρ-meson twist-2 distribution amplitudes: transverse (solid line) and longitudinal (dashed) projections. The third line is distribution amplitude at asymptotic scale.

The distribution amplitudes of the pion and the real photon calculated in the instanton model in the chiral limit may be cast in a closed form. It is convenient to introduce notations for the integration variables ($\overline{x} = 1 - x$)

$$\begin{aligned} u_+ &= u - i\lambda x, \quad u_- = u + i\lambda\overline{x}, \quad M_\pm = M\left(u_\pm\right), \\ D(u) &= u + M^2(u), \quad D_\pm = D\left(u_\pm\right). \end{aligned}$$

Then one gets the expressions

$$\phi_\pi^A\left(x\right) = \frac{1}{f_\pi^2}\frac{N_c}{4\pi^2}\int_0^\infty du\int_{-\infty}^\infty \frac{d\lambda}{2\pi}\frac{f_+f_-}{D_+D_-}\left(xM_- + \overline{x}M_+\right), \tag{9}$$

$$\begin{aligned} \phi_{\perp\gamma}\left(x, q^2 = 0\right) = \frac{1}{\left|\langle\bar{q}q\rangle\right|\chi_{\mathrm{m}}}\frac{N_c}{4\pi^2}\left[\Theta(\overline{x}x)\int_0^\infty du\frac{M\left(u\right)}{D\left(u\right)} - \right. \\ \left. - \int_0^\infty du\int_{-\infty}^\infty \frac{d\lambda}{2\pi}\frac{M_+M_-}{D_+D_-}M^{(1)}\left(u_+, u_-\right)\right]. \end{aligned} \tag{10}$$

$$\phi_{\parallel\gamma}\left(x, q^2 = 0\right) = \Theta(\overline{x}x). \tag{11}$$

The DAs are scale dependent quantities and the above expressions correspond to the low momentum scale μ_0 typical for the instanton model. For the instanton model it is estimated as $\mu_0 = 530$ MeV [11]. The parameters entering normalization coefficients are given by

$$\langle 0 \left| \overline{q} q \right| 0 \rangle^{\text{inst}} \Big|_{1\text{GeV}} = -(0.24 \text{ GeV})^3, \quad \chi_{\text{m}}^{\text{inst}} \big|_{1\text{GeV}} = 2.73 \quad \text{GeV}^{-2}.$$

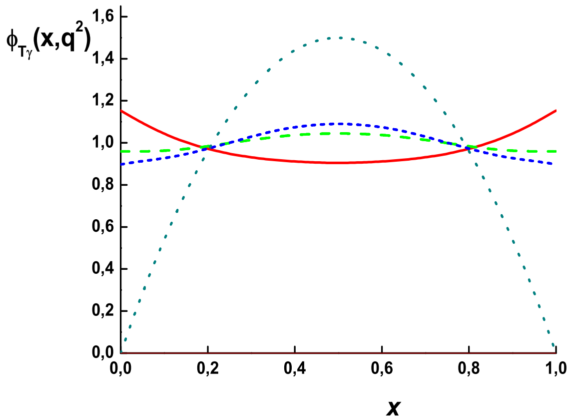

FIG. 3. Dependence of the twist-2 tensor component of the photon DA on transverse momentum squared ($q^2 = 0.25$ GeV^2 solid line, $q^2 = 0$ GeV^2 dashed line, $q^2 = -0.09$ GeV^2 short-dashed line, asymptotic DA - dotted line) given at the quark model scale.

The results of calculations are shown in Figs. 1-4. They correspond to low momentum scale μ_0 and need to be evolved to higher momenta scale in order to compare with experimentally available information. The DA at asymptotic scales $\mu_{\text{as}} = \infty$ is also presented.

4. Conclusions

The instanton model of QCD vacuum is realistic tool to get nonperturbative properties of hadrons in terms of parameters characterizing the vacuum. All hadron DAs are suppressed at the bound of kinematical interval due to localized wave function of hadrons, while photon DAs are not zero there. By applying the QCD evolution the photon DAs become immediately

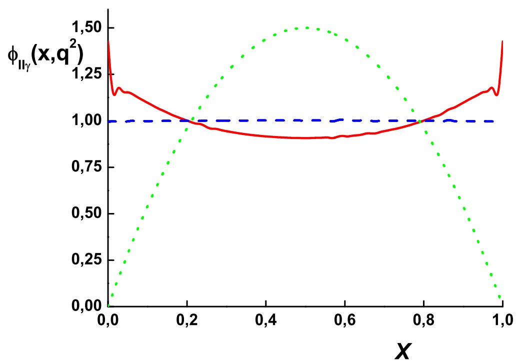

FIG. 4: Same as Fig. 3 for the twist-2 vector component of the photon DA.

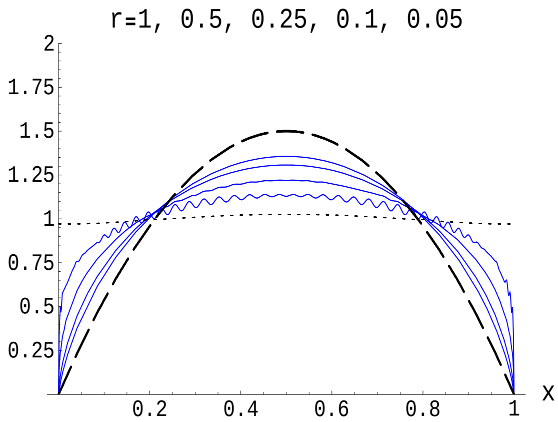

FIG. 5. The LO ERBL evolution of the nonlocal model predictions for the leading-twist *tensor* projection of the real photon DA $\phi^{(t)}_{\perp\gamma}(x,q^2)$. The dashed lines show the asymptotic DA, $6x(1-x)$. Initial conditions, indicated by dotted lines, are evaluated in the nonlocal quark model at the initial scale $\mu^{\rm inst}=530$ MeV. The solid lines correspond to evolved DA'a at scales $Q=1$, 2.4, 10, and 1000 GeV. The corresponding values of the evolution ratio r are given in the figures.

zero at the edge points of x-interval. Nevertheless, the photon DAs are always wider than asymptotic distribution. The first experimental results on direct measurements of pion and photon DAs are discussed in [15, 16].

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