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Intermittency at QGP→Hadron phase transition within generalized Ginzburg-Landau model in the squeezed state representation

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The squeezed state mechanism is used in the framework of the Ginzburg-Landau model for description of the parton-hadron phase transition in processes with high-energy densities. Normalized factorial moments are studied as functions of the bin width of the phase space at different squeezing parameters. Intermittency and scaling are investigated in the squeezed state representation at first-order phase transition.

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1. Introduction

One of the primary motivations of the study of high-energy heavy-ion collisions is to investigate the properties of quark-gluon system at extremely high temperature and high density. Such system may be in the state of quark-gluon plasma (QGP) [1]. As expected the system will undergo a quark-hadron phase transition (PT) [2] with the expanding and cooling. We can relate the coordinate variables of an expanding system undergoing PT to the kinematical variables, in terms of which the produced hadrons are measured, and z will be used to denote set of necessary variables as a whole [3, 4].

One of the theoretical aims is to find a signal about the PT. The order of a PT is sensitive to the involved approximations and has far-reaching phenomenological consequences. The experimental consequences of a first-order transition make it relatively easy to see, especially if the plasma "explodes" into the hadronic phase. A second-order transition, lacking a jump in the energy density, may be less easy to see experimentally. At present time no clear signal of the phase transition is known, although a number of signals has been proposed, and the quantities measured so far can usually be modeled by both a hot hadron gas and a quark-gluon plasma.

Lattice gauge calculations indicate that for two flavors the PT is most likely of the second order [5, 6]. When strange quarks are included, it may become a weak first-order PT [7]. Therefore we need to take into consideration the possibility both first- and second-order PT.

The multiplicity fluctuations of hadrons produced in high-energy heavy-ion collisions can be used as a measure of whether a quark-gluon system has undergone a phase transition (PT) [8] because the fluctuations are large near critical points for statistical systems. Thus the study of fluctuations in the process might reveal some features for the PT which can be tested in heavy-ion collisions.

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There are a number of efforts to apply the coherent state representation (P-representation) [9, 10] for investigation of the multiplicity fluctuations as a phenomenological manifestation of quark-hadron PT in the framework of the Ginzburg-Landau (GL) formalism both for the second-order [3, 4] and first-order [11–13] PT. One of the differences between second- and first-order phase transition is a hysteresis phenomenon which is peculiar for the first-order phase transitions only [11, 12]. In both cases multiplicity distribution of the hadrons without PT was supposed to be Poisson one that resulted in scaling behaviour of the factorial moments. The scaling exponent ν appears to be equal 1.305 for second-order PT [3] and $1.32 < \nu < 1.33$ for the generalized GL model with first-order PT [12]. These results disagree with old experimental data on heavy-ion collisions ($\nu = 1.459 \pm 0.021$ [14]) that is evidence of absence of the QGP-hadron PT in these experiments.

In present paper QGP is described phenomenologically and a transition from quarks to hadrons is considered as phase transition. We demonstrate this on an example of the generalized GL theory for phase transition from QGP to hadrons. Considering multiplicity distribution as squeezed state one [15] in the absence of PT we have generalized GL approach for investigation of the multiplicity fluctuations at phase transition from QGP to hadrons. Since the multiplicity fluctuations exhibit intermittent and scaling behaviour which is observed in a large number of experiments, we investigate conditions of appearance of these effects depending on the parameters of GL theory.

2. Intermittency and scaling at QGP→Hadron phase transition

One criterion for the order of the phase transition is given by Landau's theory [16]. It consists in an expansion of the free energy in powers of the order parameter. The allowed terms in this expansion are further selected by symmetry arguments. Phase transitions can be classified according to the transformation behavior of their order parameters under a symmetry transformation.

Within generalized Ginzburg-Landau (GL) model the free energy of the system [12] is

$$F[\psi] = \int dz \{a|\psi(z)|^2 + b|\psi(z)|^4 + f|\psi|^6 + c|\partial\psi/\partial z|^2\}, \tag{1}$$

where $\psi(z)$ is introduced to serve as a complex order parameter.

Then the hadron multiplicity distribution can be given by the functional integral of the type [17]

$$P_n = Z^{-1} \int D\psi P_n^0 e^{-F[\psi]}, \tag{2}$$

here $Z = \int D\psi e^{-F[\psi]}$. Thus the probability of having a large n in volume V is controled by deviation of ψ from ψ_0 (minimum of the GL potential) as specified by the thermodynamical factor $e^{-F[\psi]}$.

The GL free energy density may be written in the form [13] at $f \neq 0$ without consideration of the kinetic term [21] $c|\partial\psi/\partial z|^2$

$$\mathscr{F}[t] = \sqrt{\frac{a^3}{f}}\, t\, h(t), \tag{3}$$

where

$$h(t) = 1 - 2(1+g)t + t^2, \quad t = \sqrt{\frac{f}{a}}\, |\psi(z)|^2, \quad g = -\left(1 + \frac{b}{2\sqrt{a\,f}}\right). \tag{4}$$

Since $h(t)$ has two real roots when g is positive, the minimum jumps from $t = 0$ to a value between two roots when

$$b < -2\sqrt{a\,f} \tag{5}$$

for a and f both positive. This is manifestation of the first order PT [13]. The multiplicity distribution after the phase transition in the squeezed state representation is

$$\begin{aligned} P_n =& \frac{\tanh^n(r)}{2\,Z\,\cosh(r)} \int_0^{2\pi} d\phi \sum_{k=0}^{n/2} \sum_{l=0}^{n/2} (-1)^{k+l} \frac{n!\,(2k-1)!!\,(2l-1)!!}{(2k)!\,(2l)!\,(n-2k)!\,(n-2l)!} \left(\frac{2x}{a}\right)^{(n-k-l)} \\ & \times F_1^{n-2k}(r,\phi,\vartheta)(F_1^*)^{n-2l}(r,\phi,\vartheta) \int_0^\infty dt\, t^{(n-k-l)} \exp\Big\{-x\,t\,\Big[h(t) - F_2(r,\phi,\vartheta)/a\Big]\Big\}, \end{aligned} \tag{6}$$

where the normalized factor Z is equal

$$Z = \frac{1}{2} \int_0^{2\pi} d\phi \int_0^\infty dt\; e^{-x\,t\,h(t)}, \tag{7}$$

$F_1(r,\phi,\vartheta), F_2(r,\phi,\vartheta)$ are functions of the parameters r, ϕ, ϑ and in case of coherent squeezed states (CSS) [15] are equal to

$$F_1(r,\phi,\vartheta) = \frac{\cosh(r) e^{i(\phi-\vartheta/2)} + \sinh(r) e^{-i(\phi-\vartheta/2)}}{\sqrt{\sinh(2r)}}, \tag{8a}$$

$$F_2(r,\phi,\vartheta) = \cosh(2r)[\tanh(r)\cos(2\phi-\vartheta) - 1] + \sinh(2r)[\tanh(r) - \cos(2\phi-\vartheta)] \tag{8b}$$

and for scaling squeezed states (SSS) [15] are

$$F_1(r,\phi,\vartheta) = \frac{e^{i(\phi-\vartheta/2)}}{\sqrt{\sinh(2r)}}, \tag{9a}$$

$$F_2(r,\phi,\vartheta) = \tanh(r)\cos(2\phi-\vartheta) - 1, \tag{9b}$$

$$x = \delta^d \sqrt{\frac{a^3}{f}}. \tag{10}$$

Here we identify $V = \delta^d$ and regard that t is constant in every bin width δ. The obtained expression for P_n (6) will be essential at analysis phenomenon of intermittency.

One of the effective way to manifest the nature of the multiplicity fluctuations in high-energy collisions is to examine the dependence of the normalized factorial moments F_q [18, 19]

$$F_q = \frac{\langle n(n-1)\cdots(n-q+1)\rangle}{\langle n\rangle^q} = \frac{f_q}{f_1^q} \tag{11}$$

on the bin width δ in rapidity. Here $f_q = \langle n\,(n\,-\,1\,)\cdot\;\cdot\;\cdot\,(n-q+1)\,\rangle$, n is the number of hadrons detected in δ in an event, and the average are taken over all events. The multiplicity fluctuations can exhibit intermittency behaviour which is manifested by power-law behaviour of F_q on δ [18]

$$F_q \propto \delta^{-\varphi_q}, \tag{12}$$

where φ_q is referred to as the intermittency index. In this paper we examine whether (12) is valid under taking into account PT. Since

$$f_q = \sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n, \tag{13}$$

using (6) and (1) we obtain the next explicit form of f_q

$$\begin{aligned} f_q =& \frac{(\sinh r)^{2q}}{2\,Z} \int_0^{2\pi} d\phi \int_0^{\infty} dt\; e^{-x\,t\,h(t)} \sum_{n=0}^{q} \left(\frac{q!}{n!}\right)^2 \frac{(2\,\tanh r)^{-n}}{(q-n)!} \\ & \times \left| H_n \left(\sqrt{\frac{x\,t}{a}} \left[F_1(r,\phi,\vartheta) \cosh r - F_1^*(r,\phi,\vartheta) \sinh r \right] \right) \right|^2 . \end{aligned} \tag{14}$$

Obtained expression we can represent as

$$f_q = \frac{J_q}{J_0}, \tag{15}$$

where in case CSS

$$J_q = \int_0^{\infty} dt\, e^{-x\,t\,h(t)} \sum_{n=0}^{q} \frac{(q!)^2}{(q-n)!} \sum_{k=0}^{n/2} \left(\frac{(2k-1)!!}{(2k)!(n-2k)!} \right)^2 \left(\frac{x\,t}{a} \right)^{n-2k} (\cosh r)^{2k} (\sinh r)^{2(q-n+k)} \tag{16}$$

and for SSS

$$\begin{aligned} J_q =& \int_0^{\infty} dt\, e^{-x\,t\,h(t)} \sum_{n=0}^{q} \frac{(q!)^2}{(q-n)!} \sum_{k=0}^{n/2} \sum_{l=0}^{n/2} \frac{(2k-1)!!\,(2l-1)!!}{(2k)!\,(2l)!} \left(\frac{x\,t}{a} \right)^{n-k-l} \\ & \times \sum_{j=0}^{n-2l} \frac{(\sinh(r))^{2(q-n+l+j)}\; (\cosh(r))^{2(n-l-j)}}{j!\,(l-k+j)!\,(n-k-l-j)!\,(n-2l-j)!} . \end{aligned} \tag{17}$$

Then according to (11),(15) the normalized factorial moments F_q have the next form

$$F_q = J_q J_1^{-q} J_0^{q-1} \tag{18}$$

and depend on four parameters: a, x, g, r. Obviously that the normalized factorial moments have an explicit dependence on a GL model parameter a and on an additional squeeze factor r in contrast to description of QGP$\leftrightarrow$Hadron phase transition on the base of the coherent state formalism within generalized Ginzburg-Landau model [13].

If the local slope of $\ln F_q$ vs $\ln F_2$ is approximately constant then we would have the scaling behaviour (Ochs-Wosiek scaling law) [20]

$$F_q \propto {F_2}^{\beta_q}, \tag{19}$$

which is valid for intermittent systems [19]. The slopes β_q are well fitted by the formula [4]

$$\beta_q = (q-1)^{\nu}, \tag{20}$$

where ν is a scaling exponent. Mean values of the scaling exponent and its standard deviation at different parameters of the generalized GL model for CSS and SSS are represented in the Tab.1 and Tab.2 correspondingly for the same values of x, g as in [13] and at the next values of the parameters: $a = 2.25, r = 0.1, 1, 9$.

Table 1. Mean values of the scaling exponent and its standard deviation at different parameters of the generalized GL model for CSS.

g	0.0			0.2		
r	0.1	1.0	9.0	0.1	1.0	9.0
$\langle\nu\rangle$	1.472	1.295	1.322	1.512	1.301	1.322
$\delta\nu$	0.153	0.018	0.0	0.165	0.022	0.0
$\frac{\delta\nu}{\langle\nu\rangle}100\%$	10.4%	1.4%	0%	10.9%	1.7%	0%
g	0.4			0.6		
r	0.1	1.0	9.0	0.1	1.0	9.0
$\langle\nu\rangle$	1.570	1.309	1.322	1.650	1.318	1.322
$\delta\nu$	0.186	0.028	0.0	0.220	0.034	0.0
$\frac{\delta\nu}{\langle\nu\rangle}100\%$	11.8%	2.1%	0%	13.3%	2.6%	0%

Table 2. Mean values of the scaling exponent and its standard deviation at different parameters of the generalized GL model for SSS.

g	0.0			0.2		
r	0.1	1.0	9.0	0.1	1.0	9.0
$\langle\nu\rangle$	1.413	1.277	1.316	1.448	1.279	1.315
$\delta\nu$	0.138	0.006	0.003	0.149	0.007	0.004
$\frac{\delta\nu}{\langle\nu\rangle}100\%$	9.8%	0.5%	0.2%	10.3%	0.5%	0.3%
g	0.4			0.6		
r	0.1	1.0	9.0	0.1	1.0	9.0
$\langle\nu\rangle$	1.495	1.281	1.314	1.556	1.284	1.314
$\delta\nu$	0.168	0.009	0.004	0.194	0.011	0.004
$\frac{\delta\nu}{\langle\nu\rangle}100\%$	11.2%	0.7%	0.3%	12.5%	0.9%	0.3%

3. Conclusion

From obtained data we can conclude: 1) Ochs-Wosiek scaling law is not valid for both CSS and SSS if the parameter a is an order greater than the squeeze factor at the least; 2) we have the scaling behaviour of the normalized factorial moments at noticeable squeezing effect: $r \geq 1.0$, moreover the standard deviation of the scaling exponent decreases as squeeze factor increases; 3) in case CSS Ochs-Wosiek scaling law is valid with practically zero error for different values of the parameter g at large value of the squeeze factor $r = 9.0$; 4) increasing of the scaling exponent mean values $\langle\nu\rangle$ form only percent fraction with subsequent increasing of the squeeze factor ($r \gg 9.0$).

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References

[1] Jean-Paul Blaizot et al., Quark-Gluon Plasma 3, edited by Rudolph C. Hwa and Xin-Nian Wang, (World Scientific Publishing, USA, NJ, 2004, 788 pp.)
[2] Hildegard Meyer-Ortmanns, Reviews of Modern Physics **68**, No. 2, (1996), 473.
[3] Hwa R. C., Phys. Rev. **D47**, (1993), 2773.
[4] Hwa R. C., Nazirov M. T., Phys. Rev. Lett. **69**, (1992), 741.
[5] Gottlieb S., Nucl. Phys. **B20**, (1991), 247.
[6] Gausterer H., Sanielevici S., Phys. Lett. **B 209**, (1988), 533.
[7] Wilzcek F., Int. J. Mod. Phys. **A7**, (1992), 3911.
[8] Bialas A., Hwa R. C., Phys. Lett. **B 253**, (1991), 436.
[9] D.F. Walls, G.J. Milburn, Quantum Optics, (Springer-Verlag, NY., USA, 1995)
[10] Botke J. C., Scalapino D. J., Sugar R.L., Phys. Rev. **D9**, (1974), 813.
[11] Babichev L. F., Klenitsky D. V., Kuvshinov V. I., Proceed. 3d Int. Seminar "Nonlinear Phenomena in Complex Systems" (14-16 February, 1994, Polatsk, Belarus), Minsk, Institute of Physics, 1995, 149.
[12] Babichev L. F., Klenitsky D. V., Kuvshinov V. I., Phys. Lett. **B 345**, (1995), 269.
[13] Hwa R. C., Phys. Rev. **C50**, (1994), 383.
[14] Jain P. L. et al., Phys. Lett. **B 236**, (1990), 219; Phys. Rev. **C44**, (1991), 854; Phys. Rev. **C48**, (1993), 517.
[15] Hirota O., *Squeezed light* (Elsevier, Tokyo, 1992).
[16] Landau, L. D., and E. M. Lifshitz, 1958, Statistical Physics (Pergamon, Oxford).
[17] Scalapino D. J., Sugar R. L., Phys. Rev. **D8**, (1973), 2284.
[18] De Wolf E. A., Dremin I. M., Kittel W., Phys. Rep. **270**, (1996), 1.
[19] Bialas A., Peschanski R., Nucl. Phys **B273**, (1986), 703; Nucl. Phys **B308**, (1988), 857.
[20] Ochs W., Wosiek J., Phys.Lett. **B214**, (1988), 617.

[21] In future we will regard that $|\psi(z)|$ is constant in every bin width δ.