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# Sungrazing comets: Properties of nuclei and in situ detectability of cometary ions. Part 1

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A one dimensional sublimation model for cometary nuclei is used to derive size limits for the nuclei of sungrazing comets. For the case that sublimation alone is sufficient for destruction, the model yields an upper size limit as a function of nuclear density.

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## 1. Introduction

Cometary nuclei cannot be spatially resolved from earth and whenever they come close to the sun they are surrounded by a coma which is several magnitudes brighter than the nucleus.

Other methods to estimate some properties of the cometary nucleus like size and albedo are remote observations of comets far from the sun [1] and interpretation of properties of the coma in observations at high spatial resolution [2]. These observations do not reveal any information about the density, material strength or internal structure of comets.

Additional properties of cometary nuclei are revealed indirectly when comets are disrupted.

In this paper, we discuss the destruction of sungrazing comets, focussing on the about 300 comets recently detected by the SOlar and Heliospheric Observatory (SOHO). The Large Angle and Spectrometric COronograph (LASCO) on SOHO, which is sensitive to still fainter objects, detected about 300 sungrazers between January 1996 and June 2001.

None of the sungrazers discovered from space was observed after perihelion. The perihelia of most of them are between one and two solar radii, so they did not fall into the sun. The only available size estimate of a sungrazer observed by SOHO is from the Ly $\alpha$ intensity of C/1996 Y1 measured by the UltraViolet Coronograph Spectrometer on SOHO [3].

We assume that the sungrazers observed from spacecraft are destroyed completely during their passage near the sun. One might argue that a comet which is depleted of its volatiles or covered by a refractory crust during its perihelion passage might be difficult to detect post-perihelion even if it survives its perihelion passage.

This work is focused mainly on the question of what can be inferred from the complete destruction. Two processes can contribute to the destruction of the sungrazing comets detected by SOHO: sublimation and disruption. The size of the surface layer which sublimates during the perihelion passage of a sungrazing comet was first estimated analytically by Huebner [4], who equated the energy sublimated by the sungrazer with the solar input energy. A more detailed numerical model which included thermal emission of the nucleus and heat conduction into the nucleus was published by Weissman [5]. Both models result in a few tens of meters for the size of the sublimating layer. Disruption mechanisms have so far not been discussed in the context of the destruction of sungrazing comets.

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We calculate the maximum size of a sungrazer which is destroyed by sublimation alone with a numerical model similar to that [5]. A nucleus of pure water ice is assumed. We derive a semi-analytical approximation from our model which fits the numerical results from [5] and the present work well. The implications of the porous nature of cometary water ice on the thickness of the sublimating layer are discussed.

## 2. Model description

The model is restricted to the nucleus and does not include the coma. We assume that the nucleus is spherical and rotating suciently fast that all physical quantities are radially symmetric. The chemical composition is assumed to be 100 % $H_2O$ ice. At the surface, the energy balance is expressed by

$$\frac{P_{sun}(1-A)}{16\pi d^2} = \sigma T_s^4 + Z(T_s)\cdot L(T_s) - F_s, \tag{1}$$

$$Z(T) = p(T)\sqrt{\mu/(2\pi T k)}, \tag{2}$$

where $P_{sun} \approx 3.83 \cdot 10^{26}$ W is the power of the sun, $A$ is the cometary albedo, $d$ is the heliocentric distance, $\sigma$ is the Stefan-Boltzmann-constant, $T_s$ is the surface temperature, $Z(T)$ is the sublimation flux in $kg\, m^{-2}s^{-1}$, $L(T) = -582\, T + 2.62\cdot 10^6$ is the latent heat of sublimation in $J\, kg^{-1}$, $-F_s$ is the net heat conduction flux towards the interior of the nucleus, $p(T)$ is the vapor pressure in thermodynamical equilibrium, $\mu$ is the mass of one molecule and k is the Boltzmann constant. With an initial temperature distribution in the nucleus (the nucleus is assumed to be at its radiative equilibrium temperature far from the sun) and the initial heliocentric distance, this equation provides $F_s$.

The surface temperature depends on the heat conduction process in the interior of the nucleus. In order to simulate this process, the nucleus is divided into l radial layers of equal volume (see Fig. 1; temperatures are cell-centered while heat fluxes are defined at cell edges). The heat flux $F_i$ between the layers $i$ and $i+1$ is calculated from

$$F_i = -K(T(R_i))\frac{T_{i+1/2} - T_{i-1/2}}{\Delta r_i}, \tag{3}$$

where $T(R_i)$ is the temperature interpolated at the position $R_i$ corresponding to $F_i$, $\Delta r_i$ is the distance between the positions of $T_{i-1/2}$ and $T_{i+1/2}$, and $K(T)$ is the heat conductivity of the ice in $WK^{-1}m^{-1}$. The heat conductivity is not very well known for cometary ice and is one of the variables in the model. The temperature at the layer boundaries and the $\Delta r_i$ are calculated by linear interpolation:

$$T(R_i) = T_{i-1/2} + \frac{T_{i+1/2} - T_{i-1/2}}{R_{i+1} - R_{i-1}}(R_i - R_{i-1}) \tag{4}$$

and

$$\Delta r_i = \frac{1}{2}\cdot (R_{i+1} - R_{i-1}). \tag{5}$$

The upper boundary flux $F_l$ is equal to $F_s$, the lower $F_0 = 0$ because of radial symmetry. The heat fluxes allow to simulate the heat diffusion process by calculating the time derivative of the temperature in each layer, obtained from energy conservation:

$$\dot{T}_{i-1/2} = \frac{-4\pi R_i^2 \cdot F_i + 4\pi R_{i-1}^2 \cdot F_{i-1}}{V_i \cdot \varrho \cdot C_p(T_{i-1/2})}, \tag{6}$$

where $V_i$ is the volume of layer $i$, $\varrho$ is nuclear density and $C_p(T) = 16.1 \cdot T^{0.871} - 39.2$ is the heat capacity of ice in $J\,kg^{-1}K^{-1}$.

Connecting this with an orbit integrating routine, the model provides among other quantities the nucleus size $R$, the sublimation rate $\dot{m} \equiv -Z \cdot 4\pi R^2$, and the radial temperature profile of the nucleus as a function of time or true anomaly.

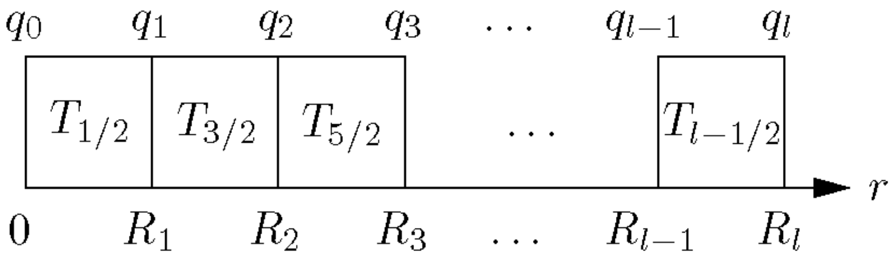


FIG. 1: Layer geometry.

## 3. Destruction by sublimation alone

In this section, we calculate the maximum size of a comet destroyed by sublimation alone as a function of the parameters $\varrho$, perihelion distance $q$ and $A$ which describe the comet in our one dimensional model. Other disruption mechanisms will be discussed in later sections.

### 3.1. Thickness of the sublimating surface layer of a sungrazer

We now use our model to calculate the thickness of the layer which will sublimate during the perihelion passage of a sungrazer. We will consider crystalline and porous water ice.

The thickness of the sublimated layer does not depend significantly on the radius of the comet. This can be seen from

$$\frac{dm}{dt} = 4\pi R^2 \varrho \frac{dR}{dt} = Z 4\pi R^2, \tag{7}$$

or

$$\frac{dR}{dt} = \frac{Z}{\varrho}, \tag{8}$$

where $\frac{dm}{dt}$ is the sublimation rate, $Z$ is the sublimation flux, and $R$ is the radius of the cometary nucleus. The sublimation flux and therefore the change in radius do not depend explicitly on the size of the nucleus. There is a small implicit dependence: A very small nucleus may be heated up internally very rapidly and then heat conduction cannot transport energy inward from the surface as efficiently as for a larger nucleus. Since our model as well as previous sublimation models [5, 6] show that comets are heated up to a depth of at most a few meters, this effect can be neglected for all practical purposes.

Fig. 2 shows the thickness of the sublimated layer for a sungrazer consisting of compact water ice ($\varrho = 931 kg/m^3$ and heat conductivity of crystalline ice as shown in Fig. 3) as a function of heliocentric distance. The sublimation during a complete orbit is considered. The upper size limit derived this way is too high. The disappearance of the comets implies that they do not reach a heliocentric distance of more than a few solar radii post perihelion. However, the difference is not very large. For a typical sungrazer, only 20-25 % of the radius of the sublimated layer evaporates post perihelion at a heliocentric distance of more than 3 solar radii. Since the maximum heliocentric distance a sungrazer can reach after perihelion without being discovered is hard to define, we conservatively define our upper size limits as the layer which evaporates during one full orbit.

A comparison of Fig. 2 with the results of the similar model of Weissman [5] shows a maximum difference of $\approx 7\%$. While this shows the excellent agreement between the two models, it is not an estimate of the error of the models, because both use the assumption of a spherical nucleus composed of pure water ice.

The cometary $H_2O$ ice may be porous and not crystalline which affects two quantities in the model: heat conductivity $K$ and density $\varrho$. Heat conductivity is expected to be much lower in porous ice than in crystalline ice. Fig. 4 shows results of our model for various values of $K$. The variation of the thickness of the sublimated layer with K is less than 3%. Therefore the choice of $K$ is uncritical as long as one is interested in the total sublimated mass only. In the model we use $K = 0.15\,W/(Km)$.

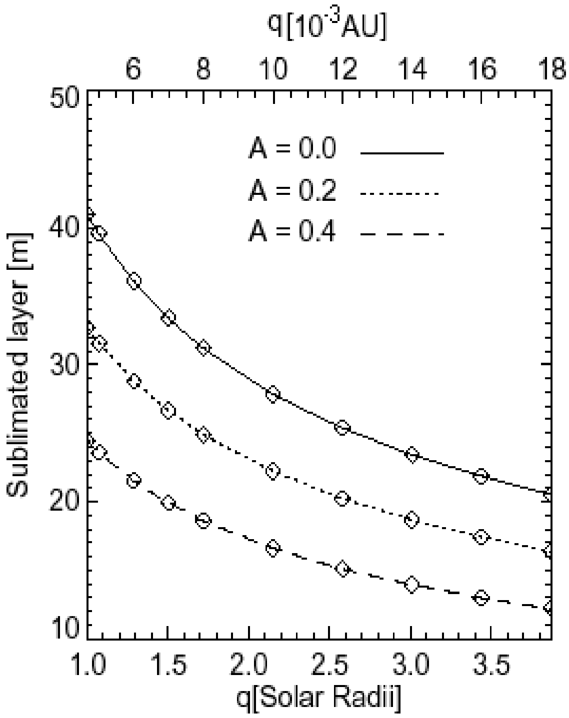


FIG. 2. Thickness $\Delta R$ of the layer sublimated from a sungrazing nucleus of compact $H_2O$ ice during the complete perihelion passage.

The density of porous ice is also lower than that of crystalline ice. Here we use the density of $600\,kg/m^3$ which Asphaug and Benz [7] derived for comet D/Shoemaker-Levy 9.

Fig. 5 shows the thickness of the sublimated layer for a comet which consists of porous ice. The values are about 50% higher than for the comet made of crystalline ice and the maximum radius is about 60 m.

### 3.2. Semi-analytical approximation

Comparing the radiation and the sublimation terms in Eq. 1 (Fig. 6) shows that whenever the sublimation process becomes important, it is orders of magnitude higher than the radiation. If one is interested in the destruction of sungrazers by sublimation, therefore mainly in the quantity $\dot{m}$, the radiation can be neglected.

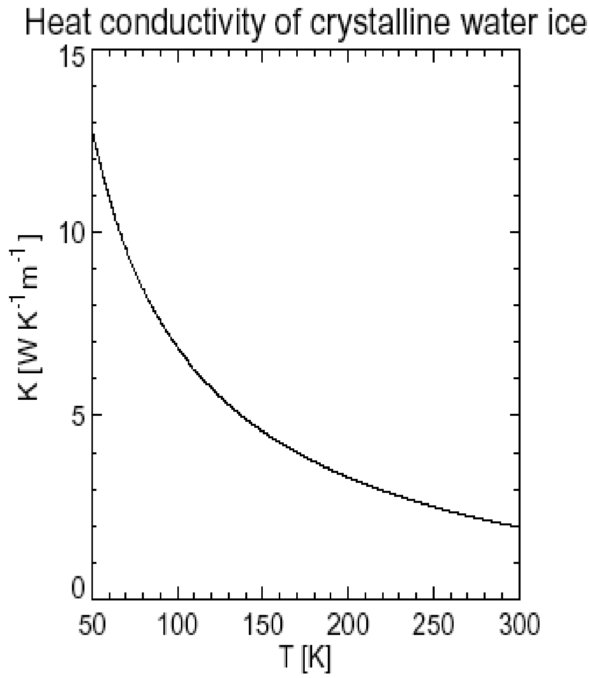


FIG. 3. Heat conductivity of crystalline ice as a function of temperature. After Hobbs [8] and Bosnjakovic [9].

Model calculations with sungrazers showed that at small heliocentric distances $\approx 85\%$ of the solar radiation energy goes into sublimation (Fig. 7). Since only a negligible part of the sublimation takes place at large heliocentric distances where sublimation is low and thermal radiation becomes important, we can simplify Eq. 1 to

$$\frac{P_{sun}(1-A)}{16\pi d^2} 0.85 = Z \cdot L \tag{9}$$

For $H_2O$, the latent heat is nearly independent of the temperature, allowing to consider $L$ as a constant. Replacing $Z$ by $-\dot{m}/(4\pi R^2)$ and solving for $\dot{m}$ yields

$$\dot{m} = -\frac{P_{sun} \cdot (1-A)}{4L \cdot d^2} \cdot R^2 \cdot 0.85 \tag{10}$$

If $\dot{m}$ is substituted with $4\pi\varrho R^2 \cdot \dot{R}$, then solving for $\dot{R}$ yields:

$$\dot{R} = -\frac{P_{sun} \cdot (1-A)}{16\pi\varrho L \cdot d^2} \cdot 0.85 \tag{11}$$

For parabolic sungrazer orbits, the time derivative of the true anomaly is $\dot{\vartheta} = \sqrt{2GM_{sun}q}/d^2$ with $G$ the gravitational constant and $M^{sun} \approx 1.989 \cdot 10^{30} kg$ the solar mass. Dividing Eq. 11 by $\dot{\vartheta}$ yields

$$\frac{dR}{d\vartheta} = -\frac{P_{sun} \cdot 0.85}{16\pi \cdot \sqrt{2GM_{sun}}} \cdot \frac{1-A}{\varrho L \sqrt{q}} \tag{12}$$

The rate of change in radius is independent of the position on the orbit. This is a consequence of the sublimated energy being a constant fraction of the solar input and is valid only when large heliocentric distances are not important. Integration of Eq. 12 over the entire orbit yields the thickness $\Delta R := \left| \int_{-\pi}^{+\pi} \frac{dR}{d\vartheta} d\vartheta \right|$ of the sublimated layer:

$$\Delta R = \frac{P_{sun} \cdot 0.85}{8 \cdot \sqrt{2GM_{sun}}} \cdot \frac{1-A}{\varrho L \sqrt{q}} \tag{13}$$

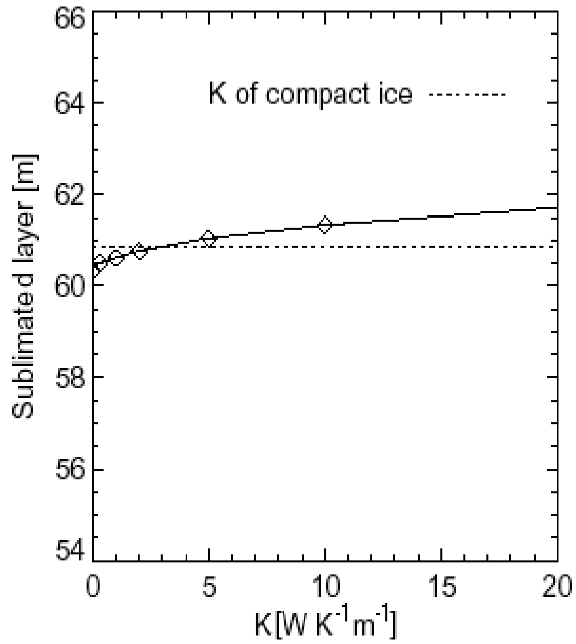


FIG. 4. Layer sublimated from a sungrazer during one perihelion passage as a function of the heat conductivity. The density is $600\,kg\,m^{-3}$, the initial radius of the comet 200 m, and the heliocentric distance at perihelion 0.005 AU ($1.1 R_{sun}$).

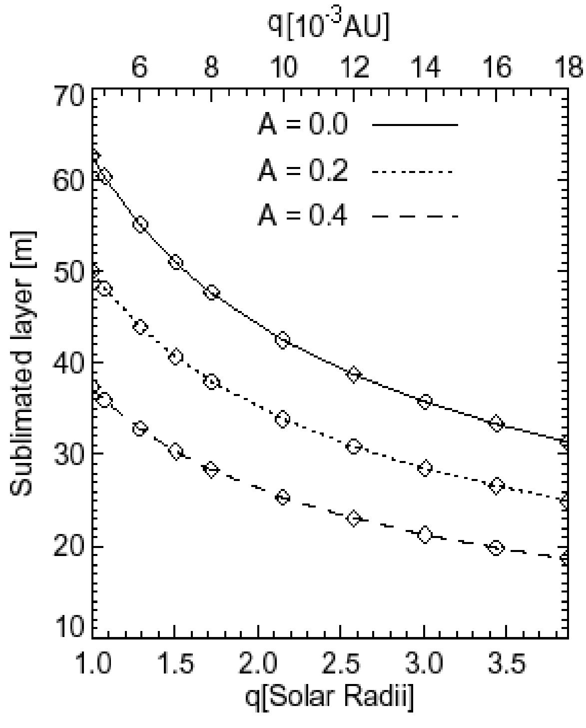


FIG. 5. Thickness $\Delta R$ of the layer sublimated from a sungrazing nucleus of porous $H_2O$ ice (density $600\,kg\,m^{-3}$, heat conductivity $0.15\,K\,W^{-1}m^{-1}$) during the complete perihelion passage.

It is important to note that $\Delta R$ is independent of $R_0$. Collecting all comet-independent numerical factors and fixing $L \approx 2.5 \cdot 10^6 Jkg^{-1}$ yields

$$\Delta R \approx 10^9 \cdot \frac{1-A}{\varrho\sqrt{q}}, \tag{14}$$

where $\Delta R$ and $q$ are in meters and $\varrho$ in $kg/m^3$. A similar result was obtained by Huebner [4].

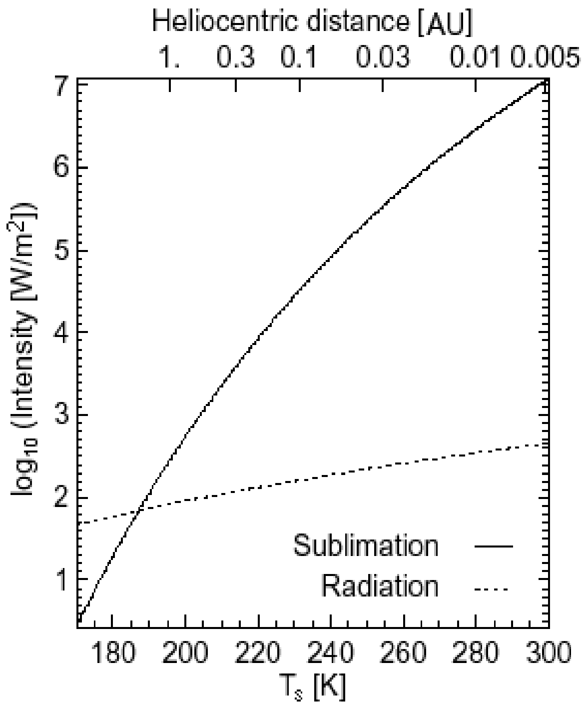


FIG. 6. Comparison of the radiation and the sublimation terms as functions of the surface temperature. The upper axis denotes the heliocentric distance of a sungrazer with perihelion at 0.005 AU.

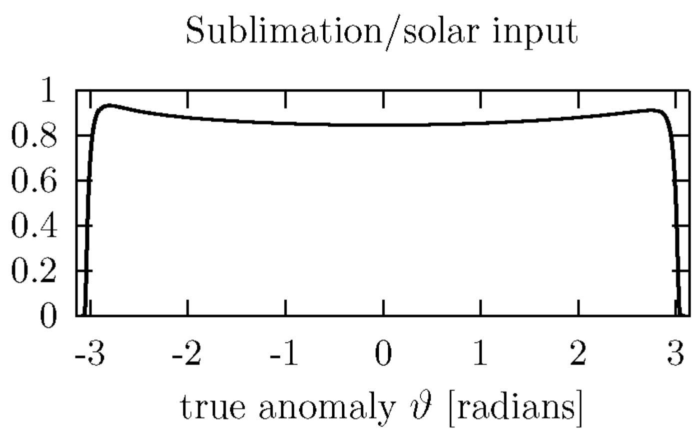


FIG. 7: Ratio of the energy lead into sublimation to the total solar radiation energy.

The comparison of the results of the numerical model with Eq. 13 shows a maximum discrepancy of 4%. Within the accuracy of the model, Eq. 13 can clearly be used to calculate $\Delta R$. The advantage of the analytical approximation for future applications is that Eq. 13 is much easier to use than the full numerical model.

## 4. Discussion

Given that we assumed that a sungrazer is destroyed by sublimation alone, its initial radius $R_0$ cannot exceed $\Delta R$. Hence Eq. 13 allows to calculate maximum initial sizes as a function of $\varrho$. $q$ is known for a given sungrazer, and $A$ can be set to zero as generally the cometary albedo is very low, resulting in an upper size limit valid for any albedo.

Considering only sungrazers but no sun-impactors implies $q \geq R_{sun}$ (solar radius, $\approx 6.96 \cdot 10^8$ m); Inserting $A = 0$ and $q = R_{sun}$ in Eq. 14 yields an upper size limit for all sungrazers with a given density:

$$R_0 \leq 3.8 \cdot 10^4 \cdot \varrho^{-1}, \tag{15}$$

where $\varrho$ is in $kg\,m^{-3}$ and $R_0$ is in m. For the density of porous ice, $\approx 600\,kg\,m^{-3}$, which was derived for comet Shoemaker-Levy 9 in Asphaug and Benz [7], the upper limit in size for the comets is $\approx 63$ m.

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