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Reaction of electromagnetic radiation for relativistic dipolar objects in the classical relativistic theory of spin

A.A. Sokolsky*
Belarusian State University,
4 Nezalejnosty Ave., 220050 Minsk, Belarus

In the framework of the classical relativistic theory of spin, the expressions for moment and force of reaction of electromagnetic radiation for objects with a charge and dipole moments are derived. It is shown that on this basis the relations directly generalize well-known Bargmann–Michel–Telegdi spin equations from the viewpoint of a consideration of the radiation self-interaction can be deduced.

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Reaction of the electromagnetic radiation caused by the magnetic moment is of interest for a wide range of problems: from evolution of pulsars up to self-polarization of particles of high energy in magnetic and electric fields. As is known, under suitable conditions [1], the behavior of relativistic particles with the intrinsic moments can be considered (including the influence of reaction of radiation) according to the classical relativistic theory of spin (CRTS) [2].

In [3], in the framework of CRTS for a relativistic particle with a charge, spin and dipole magnetic (and/or electric) moment, the expressions for the force and the moment of force of reaction of electromagnetic radiation were derived. As the first step, the regular part of the electromagnetic field created by a particle in a point of its localization was found. According to Dirac [4], this part is a half-difference of retarded and advanced fields of radiation. The second step consisted in the addition of this regular part to an external electromagnetic field and in substitution of result in the corresponding equations of motion. To obtain the expression for force of reaction, it is necessary to find not only a regular part of the field tensor but also its gradient.

In [3], "self-actions" of three kinds were taken into account in the expression for the moment of force: "charge-charge", "moment-charge" and "moment-moment", while this latter was neglected in the expression for the force. The derived 4-equations of spin look like:

$$\overset{\circ}{S}{}^{\alpha} = [\mu \times B_{ext}]^{\alpha} + [\eta \times E_{ext}]^{\alpha} + N^{\alpha}_{rad}, \tag{1}$$

where the 4-moment of force of reaction of radiation takes the form

$$N^{\alpha}_{rad} = \frac{2}{3} q \left[\eta \times \ddot{u}\right]^{\alpha} + \frac{2}{3}(\chi^2 + \alpha^2)\left(\left[S \times \overset{\circ\circ\circ}{S}\right]^{\alpha} + u_{\nu} u^{\nu} \left[\dot{S} \times S\right]^{\alpha}\right), \tag{2}$$

q is the charge, S_α is the 4-spin, $\mu_\alpha = \chi S_\alpha$ and $\eta_\beta = \alpha S_\beta$ are the magnetic and electric dipole 4-moments, u_α is the 4-velocity, $H_\alpha = \frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta} F^{\beta\gamma} u^\delta$ and $E_\alpha = F_{\alpha\beta} u^\beta$ are the 4-vectors of magnetic and electric fields, $\varepsilon_{\alpha\beta\gamma\delta}$ is the antisymmetric 4-tensor, $[a \times b]_\alpha = \varepsilon_{\alpha\beta\gamma\delta}\, a^\beta\, b^\gamma\, u^\delta$, $\overset{\circ}{a}_\beta = \left(\delta^\nu_\beta + u_\beta u^\nu\right) \dot{a}_\nu$, $\dot{a}_\beta = D a_\beta / d\tau$, τ is the proper time.

It should be noted that at $u_\alpha = const$, $\eta_\alpha = 0$ relation (2) gives the well-known [5] expression for the dissipative moment of force of reaction of radiation for a free magnetic dipole, and in

*E-mail: sokolan@tut.by

the general case (1) and (2) allow to take into account simultaneously both the reaction of radiation and the influence of relativistic orbital movement on the evolution of the spin of a dipolar object.

It is also essential that, as shown in [6], from (1) and (2) it is possible to deduce the relations generalizing the widely used Bargmann–Michel–Telegdi (BMT) spin equations [7]. A substitution of the expressions for $\dot{\vec{S}}_\alpha$ and $\overset{\circ\circ\circ}{\vec{S}}_\alpha$ calculated according to equation (1) (but without the term N^α_{rad}) is sufficient for this purpose. As a result, after corresponding transformations the relativistic spin equations with the self-action, that do not contain the derivatives of the high order, were deduced in [6]. If $\eta_\beta = 0$, they look like:

$$\dot{S}_\alpha = u_\alpha \, \dot{u}_\beta \, S^\beta + [\Omega \times S]_\alpha \,, \quad \Omega^\alpha = -\chi H^\alpha + \Omega^\alpha_{rad} \quad , \tag{3}$$

where $\Omega^\alpha_{rad} = \frac{2}{3}\chi^3 \left(\chi^2 H_\beta H^\beta + \dot{u}_\nu \dot{u}^\nu\right) [S \times H]^\alpha +$

$$+\frac{2}{3}\chi^3 \left[\overset{\circ\circ}{H} \times S\right]^\alpha - \frac{2}{3}\chi^4 \left(\overset{\circ}{H}{}^\alpha \, S^\beta H_\beta + 2H^\alpha S^\beta \, \overset{\circ}{H}_\beta\right). \tag{4}$$

Comparison with [7] shows that relations (3) and (4) generalize the BMT equations concerning a consideration of reaction of radiation and their applicability for nonuniform external electromagnetic field. In an uniform field, Ω^α_{rad} reduces to the expression:

$$\Omega^\alpha_{rad} \to \frac{2}{3}\chi^3 \left(\chi^2 H_\beta H^\beta + (q/m)^2 E_\beta E^\beta\right) [S \times H]^\alpha \,, \tag{5}$$

where m is the particle mass.

From relations (3) and (5), the 3-dimensional spin equations follow:

$$\frac{d\vec{\sigma}}{dt} = [\vec{\omega} \times \vec{\sigma}] \,, \quad \vec{\omega} = -\chi H_v + \vec{\omega}_T + \vec{\omega}_{rad}, \tag{6}$$

where $\vec{\omega}_T = \frac{\gamma^2}{(\gamma+1)c^2} \left[\frac{d\vec{v}}{dt} \times \vec{v}\right]$, $\quad \vec{v} = \frac{d\vec{r}}{dt}, \quad \gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$,

$$\vec{\omega}_{rad} = \frac{2}{3}\gamma^2\chi^3 \left(\chi^2 H_v^2 + (q/m)^2 E_v^2\right) \left[\vec{\sigma} \times \vec{H}_v\right], \quad \vec{\sigma} = \vec{s} - \frac{\gamma\vec{v}}{\gamma+1}\left(\vec{s} \cdot \frac{\vec{v}}{c}\right),$$

$$\vec{H}_v = \vec{H} + \left[\vec{E} \times \frac{\vec{v}}{c}\right] - \frac{\gamma\vec{v}}{(\gamma+1)\,c^2}\left(\vec{H} \cdot \vec{v}\right), \quad \vec{E}_v = \vec{E} - \left[\vec{H} \times \frac{\vec{v}}{c}\right] - \frac{\gamma\vec{v}}{(\gamma+1)\,c^2}\left(\vec{E} \cdot \vec{v}\right).$$

Equations (6) shows that the 3-moment of the force of radiation reaction

$$[\vec{\omega}_{rad} \times \vec{\sigma}] = \frac{2}{3}\gamma^2\chi^3 \left(\chi^2 H_v^2 + (q/m)^2 E_v^2\right) \left[\vec{\sigma} \times \left[\vec{H}_v \times \vec{\sigma}\right]\right]$$

tends to orientate 3-vector of a spin $\vec{\sigma}$ along the direction of the vector $\vec{H}_v$.

The expression for the 4-force of reaction of radiation derived in [3] looks like:

$$F^{rad}_\alpha = \frac{2}{3}q^2 \, \overset{\circ\circ}{u}_\alpha + \frac{2}{3}q \left(\left[\overset{\circ\circ\circ}{u} \times \mu\right]_\alpha + 2\left[\ddot{u} \times \dot{\mu}\right]_\alpha\right) +$$

$$+\frac{2}{3}q\left(\overset{\circ\circ\circ}{\eta}_\alpha - u_\nu u^\nu \, \overset{\circ}{\eta}_\alpha - \eta^\nu \, \ddot{u}_\nu \, \dot{u}_\alpha + u^\nu \, \dot{\eta}_\nu \, \overset{\circ\circ}{u}_\alpha\right). \tag{7}$$

The first term in (7), as it must, coincides with the expression of the 4-force of reaction of radiation for a relativistic particle with a charge [4]. For completeness of taking into account of the influence of reaction of electromagnetic radiation on orbital movement of an object (that may be essential, for example, for evolution of system of double pulsars) it is necessary to calculate the members which describe the "moment-moment" self-action and to add them to (7).

It is important that for a consideration of the influence of an external gravitational field within the limits of the general theory of relativity, by virtue of local character of "self-action", it is sufficient to understand all considered 4-dimensional relations as the general covariant ones in the Rieman space-time.

References

[1] Berestetsky V. B., Lifshitz E.M. Pitaevsky L.P. Quantum electrodynamics. M.: Nauka, 1980. P. 183.

[2] Ternov I.M., Bordovitsin V.A. // UFN. 1980. V. 132. P. 345.

[3] Sokolsky A.A.// Gravitation and electromagnetism. Minsk, BSU Press, 1988. P. 215.

[4] Dirac P.A.M.// Proc. Roy. Soc London. 1938. Vol. A 167. P. 148.

[5] Guinzburg V.L. Theoretical physics and astrophysics. M.: Nauka. 1975. P. 37.

[6] Sokolsky A.A.// Gravitation and electromagnetism. Minsk, BSU Press, 1989. P. 139.

[7] Bargmann V., Michel L., Telegdi V. L. // Phys. Rev. Lett. 1959. V.2. P. 435.