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Chaos suppression effect in intermittent modes

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We consider two-parametric piecewise continuously differentiable mapping with the bifurcation point at the boundary. It demonstrates a new type of intermittent behavior. Main statistical characteristics of these modes are calculated analytically and numerically. We show that the intermittent modes with zero Lyapunov exponent exist. We also discuss transitions chaos–weak chaos and order–weak chaos. In these transitions we observe the effect of the suppression chaos. If source chaotic mode has intermittent type it transition to weak chaos goes with reconfiguration of laminar phases. Observed intermittency has two different type laminar phases.

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1. Introduction

As well known intermittency is one of routine scenario of chaos transitions. In 1D maps it occurs in the vicinity of a saddle-node bifurcation.Intermittent mode includes elements of regular dynamics and elements with chaotic nature. First classification of different intermittent modes was originally proposed by Pomeau and Manneville [1]-[3]. They classified these instabilities as type I,II and III, according to the Floquet multipliers of the map crossing the unity circle in the complex plane at 1, at a pair complex conjugate values or at - 1, respectively. This classification is based on the assumption of map's smoothness in a bifurcation point. We can observe new types of chaos transitions in maps with violation of this property [8]-[10]. In this article we also discuss properties of a mode witch appears in the set of maps with such violation.

2. Two-parametric maps family.

If continuous map has neutral unstable fixed point at the boundary of phase space, a new type of intermittency can be observed in it. Map's asymptotic form in this point is: $x_{n+1} = x_n + x_n^\alpha$ where $\alpha > 1$.

We use two-parametric maps family for the detailed exploration of such modes. It has the following form:

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$$f(x) = \begin{cases} x + x^{\alpha} & 0 \le x < 1 \\ 4 - 2\left|x-2\right|^{\beta} & 1 \le x \le 3 \\ 4 - x + (4-x)^{\alpha} & 3 < x \le 4 \end{cases} \tag{1}$$

Phase space of this maps family is an interval of the real axis $[0,4]$.Parameters $\alpha > 1$ and β are positive. Existing of two fixed points is the feature of these maps.The first point $x = 0$ lies on the boundary of phase space. This point is neutral unstable under any values of the parameters ($\alpha > 1$ и $\beta > 0$). Its multiplier equals one.

$$\lim_{x \to +0} f'(x) = 1 + 0$$

The second fixed point lies in the interval $2 < x < 3$ and is defined by the solution of the transcendent equation:

$$4 - 2\left(x^{*} - 2\right)^{\beta} - x^{*} = 0$$

It's easy to check that fixed point is stable under condition $0 < \beta < \beta^{*}$. Under condition $\beta > \beta^{*}$ it loses stability. Its multiplier cross value -1 under parameter value $\beta = \beta^{*} \approx 0.26$. Here we can observe chaos transition by the intermittency III type.

3. Dynamical properties of the map.

3.1. Statistics of laminar phases.

The system can be found in different modes under different values of parameters. Intermittent modes have special structure. It consists of laminar phases and chaotical bursts. Its pattern weakly depends on the variation of extended parameters, therefore measure of random nature depends on statistics of laminar phases rather then its internal organization. For the evaluation of the distribution function we need to investigate the map's dynamic of laminar intervals as well as statistical properties of the reinjection mechanism.

Map's dynamics analysis in the range of small x allows us to find the length of laminar phases when we know their entry conditions. Initial coordinate x_0 has stochastic nature. Its statistical properties depend on reinjection mechanism. Distribution of value x_0 defines invariant distribution function in the range $4 - x \ll 1$. It follows from structure of the map using Frobenius-Perron equation. Thus the asymptotic form of the distribution lengths of laminar phases in the region $l \gg 1$ is :

$$dw \sim \frac{1}{l^{1+\frac{1}{(\alpha-1)\beta}}} dl = \frac{1}{l^{\gamma}} dl \tag{2}$$

Exponent $\gamma > 1$ under conditions $\alpha > 1$ and $\beta > 0$. We note that the distribution of laminar lengths has slow power falling. It is typical feature of Levy's distributions. Large-scale range power scaling of the laminar lengths distribution account for anomalous statistical properties and the divergence of its distribution moments. The results map of computer modelling of map (1) justify the existing of the such power falling $\rho(l) \sim l^{-\gamma}$. Its modes have wide spectrum of scaling exponent which depends on external parameters. In contrast to intermittent modes of I and III types with scaling exponent equals to 0.5. All the moments of the obtained distribution higher then $\frac{1}{(\alpha-1)\beta}$ are divergent. The first moment of the distribution diverges under condition $\frac{1}{(\alpha-1)\beta} \ge 1$. It means that mean laminar length converge to infinity $< l > \to \infty$.

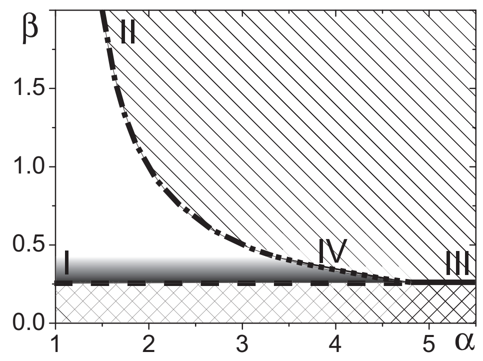

Рис. 1. Phase portrait. Parameter region with zero (marked by skew hatching) and positive (white region) Lyapunov exponent. Shew checked parameter region correspond to regular dynamics phase (Lyapunov exponent is negative).

3.2. Lyapunov exponent

Now we estimate the value of a chaotic nature map's tracks (1). To do it, we calculate Lyapunov exponent. Lyapunov exponent for one- dimensional maps is defined as:

$$\Lambda = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \ln |f'(x_n)| \qquad (3)$$

The sum in the right part contains deposits from laminar and chaotical sections. It's physically convenient to divide it and analyze it by these sections.

The deposit of laminar sections in the Lyapunov exponent is obtained by distribution of laminar lengths and randomness degree during one laminar section.

$$\Lambda_{lam} = \frac{\lambda_{lam}}{\langle l\rangle + \langle l_{chaos}\rangle}$$

where λ_{lam} - means Lyapunov exponent of one laminar phase. Its value is positive and finite under any values of parameters. Similar expression defines deposit chaotical bursts in the Lyapunov exponent. The minutiae of laminar and chaotic sections structure are not significant. Thus we can obtain the expression of total Lyapunov exponent:

$$\Lambda \sim \frac{const}{< l > + < l_{chaos} >}$$

For the I type intermittency this expression has been obtained in [7]. In certain parameter region value $< l > \to \infty$. Here Lyapunov exponent vanishes. In such systems we can observe the weak chaos mode. Anomalous properties of laminar lengths distribution generates two physically different dynamics modes. Value $\frac{1}{(\alpha-1)\beta} = 1$ is the boundary separating the chaotical modes upon ordinary chaos ($\langle l\rangle < \infty$) and weak chaos with zero Lyapunov exponent.

4. Different phase transitions. Chaos suppression effect.

Parameter (β, α) region of the two-parametric map (1) is divided into three qualitative different regions (fig .1).

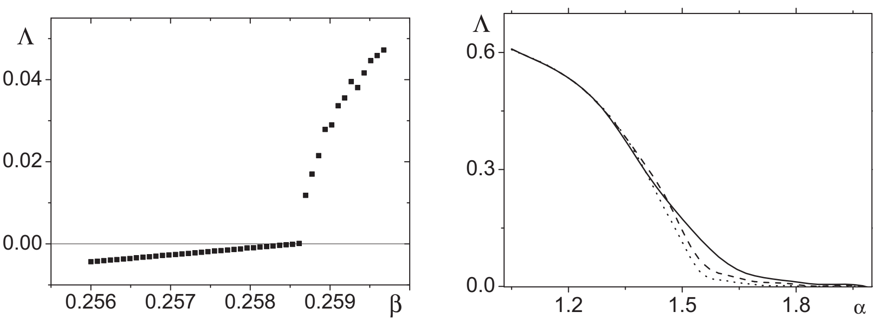

Рис. 2. Behavior of Lyapunov exponent about the bifurcation point $\beta = \beta^*$. Transition trough intermittency of III type, follows the cascade of period doubling.

4.1. Transition chaos–weak chaos.

Consequent origin of the weak chaos demonstrates effect of the chaos suppression. In the present case chaotic nature of the system's dynamic would be preserved under any values of external parameters. In contrast to ordinary mechanism of the intermittency which in extreme case it has regular mode. This effect connected with specific character of the reinjection mechanism and dynamics of laminar sections. At the result laminar phase statistics have essentially anomalous character. We can observe power scaling with exponent dependent on map's parameters.

At fig. 2 we can see qualitative change in the map's behavior under transition of value $\frac{1}{(\alpha-1)\beta}$ through 1. System demonstrates chaos behavior under $\alpha < 1.5$ and weak chaos behavior under $\alpha > 1.5$.

Transition from the region of mature chaos to the weak chaos region (line II) correspond to specifical phase transition. Lyapunov exponent can be considered like order parameter. Here continuous character of order parameter change correspond to II type phase transition.

For the confirmation of chaotic nature of zero Lyapunov modes we can evaluate topological entropy. For the class of maps under consideration it value can be directly calculated

$$h_{top} = ln(2)$$

It coincides with value of topological entropy of logistic map. In contrast to logistic map for maps family under consideration value of topological entropy does not coincide with Lyapunov exponent. Lyapunov exponent can change with variation of external parameters and even vanish to zero. Therefore result about the equality of Lyapunov exponent to topological entropy fails for this intermittent modes. This explains by nonintegrable divergence of the invariant distribution function.

4.2. Transition order–weak chaos. Reconstruction of the chaotical mode

Transition from regular behavior to mature chaos (line I) passes by scenario of III type intermittency. It occurs under multiplier of the fixed point x^* cross the value -1 under parameter $\beta = \beta^* \approx 0.26$. It justified by computer modelling of the Lyapunov exponent(fig. 3).

Usually at transition of multiplier through -1 doubling-period cascade is observed. But in the considered set of maps we observe the intermittency of III type. It is connected with the lack of smooth extremum near the fixed point. Reinjection in this map tightly bound with non-smoothness of its form at the point $x = 2$.

Рис. 3. Transition chaos–weak chaos with increasing α parameter. Under the same value $\alpha > \alpha_{crit}$ Lyapunov exponent vanished. The tendency of approaching numerical results to theoretical estimation with increasing number of iterations which used at Lyapunov exponent calculating.(continuous line - 10^6, long dotted line - 10^6, short dotted line - 10^9 iterations) is shown

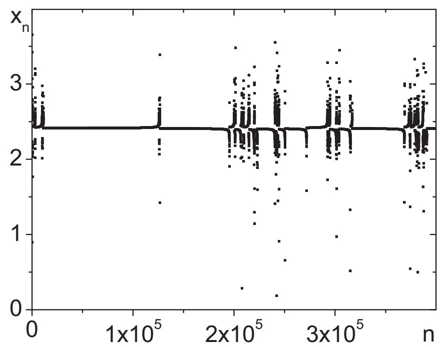

Рис. 4: Typical map's iteration diagram at transition through line I (III type intermittency).

Note that under certain values of parameters α и β there exists direct transition from regular dynamics to weak chaos without intermediate phase with positive Lyapunov exponent(line III). Boundary of this transition defines like

$$\alpha \geq 4.86$$
$$\beta \approx 0.26$$

The observed transition can't be categorized within this limits of ordinary theory.That is consists of two different type of laminar phases (fig. 4). The competition between the intermittency of III type and the intermittency of boundary point appears. Mean laminar length at neighborhood x^* decreases while parameter β increases, but under parameter value $\alpha \geq 4.86$ statistical properties of laminar phases near $x = 0$ make Lyapunov exponent vanish. It is an example of process order–weak chaos transition. General mechanism of transition of this kind is gradual change of phase space which finishes with the appearance neutral unstable point at its boundary.

Competition between the intermittencies can exist not only under transition order–weak chaos. If source chaos mode has intermittent structure, it transition to weak chaos is

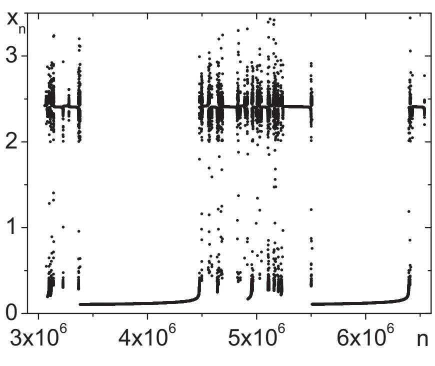

Рис. 5. Map's iteration diagram at transition through line III (transition order-weak chaos). The reconstruction of the chaotical mode is observed.

accompanied by a mode with two types of laminar sections. First of all, the second type of laminar phases appears. With parameter change a part of the second type laminar phases is gradual increases. It is finished by vanishing Lyapunov exponent. This process can be called the reconstruction of the chaotical mode.

5. Conclusion

The demonstrated chaotical properties at zero Lyapunov exponent are not a typical behavior for classical intermittent modes. In 1D maps weak chaos behavior is demonstrated only by maps with boundary bifurcation point. Weak chaos mode resembles the behavior of Hamiltonian stochastic systems, where total Lyapunov exponent also equals to zero. We can expect that maps from the considered family take important part in the investigation of Hamiltonian chaos.

Such maps can be used for modelling the processes with flicker-noise. Fourier analysis of the tracks of considered maps set shows the existence of the power divergence in their spectrums. Exponent of this divergence depends on map's parameters on a complicated way.

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