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Confinement in 2-dimensional field model due to scalar instantons.

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2-dimensional scalar field theory with Sin-Gordon potential is considered in the case of finite space region. It is shown, that confinement of external fermions is realized in the model due to instantons (tunneling transitions between classically degenerated vacua).

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Let us consider 2-dimensional scalar theory (Sin-Gordon model):

$$\mathrm{L} = \frac{1}{2}\partial_\mu\varphi\partial_\mu\varphi - \lambda(1-\cos(\rho\varphi)), \qquad \mu = 0, 1, \tag{1}$$

where $\varphi(x,t)$, λ, ρ are real, $-\frac{L}{2} \le x \le \frac{L}{2}$ and $\varphi(\frac{L}{2}) = \varphi(-\frac{L}{2})$.

This model admits instantons (non-trivial classical solutions of the field equations in imaginary time $\tau = it$) [1]:

$$\varphi^{inst}(\tau) = \pm\frac{4}{\rho}\arctan e^{(\tau-\tau_0)\rho\sqrt{\lambda}}, \tag{2}$$

where τ_0 is arbitrary parameter.

In real time instantons (2) correspond to the tunneling transitions between neighbor classically degenerated vacua $\varphi_n^{vac} = \frac{2\pi n}{\rho}$, where n is integer.

As is known, instantons lead to the confinement phenomena in certain of gauge field theories [2], [3]. Here we estimate the potential between external fermions in the model (1) by Wilson loop method [4].

Let us first of all introduce external fermions in the lagrangian (1) by means of Ukawa interaction:

$$\mathrm{L}_{int} = g\bar{\psi}\psi\varphi, \tag{3}$$

where $\bar{\psi}(x,t)$ and $\psi(x,t)$ are 2-component fermions, $\bar{\psi} = \psi^+\gamma_0$, $\gamma_0 = \sigma_1$ (Pauli matrix), g - coupling constant. Corresponding contribution in Euclidean action is

$$S_{int} = ig\int\limits_{-\frac{L}{2}}^{\frac{L}{2}} dx \int\limits_{-\frac{T}{2}}^{\frac{T}{2}} d\tau\bar{\psi}\psi\varphi, \tag{4}$$

where $T \to \infty$ is imaginary time interval, $L < \infty$ is space interval.

Let us suppose that external fermions produce Wilson loop with (imaginary) time size T_1 and space length L_1. Suppose, that $L_1 < L$, $T \to \infty$, $T_1 \to \infty$, $T_1 \ll T$. Then we can approximately write

$$\bar{\psi}(x,\tau)\psi(x,\tau) \approx \begin{cases} \delta\left(x - \frac{L_1}{2}\right) + \delta(x + \frac{L_1}{2}), \; -\frac{T_1}{2} \le \tau \le \frac{T_1}{2} \\ 0, \; \tau < -\frac{T_1}{2} \\ 0, \; \tau > +\frac{T_1}{2} \end{cases}. \tag{5}$$

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Then, action (4) takes the form:

$$S_{int} = ig \int\limits_{-\frac{T_1}{2}}^{\frac{T_1}{2}} d\tau \left[\varphi\left(\frac{L_1}{2}, \tau\right) + \varphi\left(-\frac{L_1}{2}, \tau\right) \right]. \qquad (6)$$

In order to estimate effective potential of interaction between two external fermions by means of Wilson method we can calculate the amplitude "vacuum→vacuum" in presence of external sources as functional integral (this way is described in detail in [5]):

$$< W >_\theta = \frac{1}{N} \int\limits_{all\, Q} [D\varphi] e^{-S[\varphi(x,\tau)]} e^{iQ\theta} e^{iS_{int}}, \qquad (7)$$

where Q is topological charge:

$$Q = \frac{\rho}{2\pi} \left[\varphi(\tau = +\infty) - \varphi(\tau = -\infty) \right], \quad Q(\varphi^{inst}) = \pm 1, \qquad (8)$$

θ characterizes real vacuum $\mid \theta >$ including tunneling transitions between classically vacua $\mid \varphi_n^{vac} >$:

$$\mid \theta > = \sum_{n=-\infty}^{+\infty} e^{in\theta} \mid \varphi_n^{vac} >, \qquad (9)$$

normalization factor N is amplitude "vacuum→vacuum" without external sources:

$$N = \int\limits_{all\, Q} [D\varphi] e^{-S[\varphi(x,\tau)]} e^{iQ\theta}. \qquad (10)$$

Potential of the interaction of external static fermions in dependence of the distance between them $V(L_1)$ is given by the following expression (see e.g. [5]):

$$V(L_1) = \lim_{T_1 \to \infty} \left(-\frac{1}{T_1} \ln < W >_\theta \right). \qquad (11)$$

Let us now calculate normalization factor N in gauss approximation and in approximation of dilute instanton gas [6]:

$$N = Ae^{-CLT} e^{2BLT \cos\theta e^{-S_0}}, \qquad (12)$$

where A, B, C are constant which can be in principle calculated and S_0 is instanton action [1]:

$$S_0 = S[\varphi^{inst}] = \frac{8\sqrt{\lambda}}{\rho} L. \qquad (13)$$

In order to calculate numerator in amplitude (7) which includes the integral on Wilson loop it is necessary to divide the contribution of the instantons (anti-instantons) situated inside and outside the loop. Suppose that we have n_I^{in} instantons and $n_{\bar{I}}^{in}$ anti-instantons inside the loop and n_I^{out} instantons and $n_{\bar{I}}^{out}$ anti-instantons outside the loop (we neglect by the (anti)instantons situated on the boundaries of the loop due to dilute gas approximation is used). Then (7) can be calculated by the following way

$$< W >_\theta = \frac{1}{N} e^{-CLT} \sum_{n_I^{in}, n_{\bar{I}}^{in}} \frac{(BL_1T_1e^{-S_0})^{n_I^{in}+n_{\bar{I}}^{in}}}{n_I^{in}!\, n_{\bar{I}}^{in}!} e^{i\left(n_I^{in}-n_{\bar{I}}^{in}\right)\left(\theta+\frac{4\pi g}{\sqrt{\lambda}\rho}\right)} \times$$

$$\sum_{n_I^{out},n_{\bar I}^{out}} \frac{\left(B(LT-L_1T_1)e^{-S_0}\right)^{n_I^{out}+n_{\bar I}^{out}}}{n_I^{out}!\,n_{\bar I}^{out}!} e^{i\left(n_I^{out}-n_{\bar I}^{out}\right)\theta} =$$

$$\exp\left[2BL_1T_1e^{-\frac{8\sqrt{\lambda}}{\rho}L}\left\{\cos\left(\theta+\frac{4\pi\ g}{\rho\sqrt{\lambda}}\right)-\cos\theta\right\}\right]. \tag{14}$$

Taking into account (11) we obtain

$$V(L_1) = 2BL_1e^{-\frac{8\sqrt{\lambda}}{\rho}L}\left\{\cos\theta-\cos\left(\theta+\frac{4\pi\ g}{\rho\sqrt{\lambda}}\right)\right\}. \tag{15}$$

Linear dependence on the distance L_1 between fermions means confinement. This phenomena is appeared due to instantons. Formula (15) is not applicable for shot distances.

Without taking into account of instantons (ordinary perturbative vacuum, e.g. $\varphi_0^{vac} = 0$) we have well known short-range attractive Ukawa potential in 2 dimensions:

$$V(L_1) \propto L_1 e^{-L_1\rho\sqrt{\lambda}}. \tag{16}$$

Thus main conclusion is instantons lead to confinement of the external fermions in scalar field model in 2 dimensions. It similar to the results which was obtained for gauge theories in 2 and 3 dimensions [2, 3].

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