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Dynamics of magnetospheres around nonrotating black holes in active galactic nuclei

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We develop the analytical model of accretion disk magnetosphere dynamics around supermassive nonrotating black holes in the centers of active galactic nuclei. Based on general relativistic equations of magnetohydrodynamics we find nonstationary solutions for time-dependent dynamo action in the accretion disks, spatial and temporary distribution of magnetic field. We show that there are two distinct stages of dynamo process: transient regime and steady-state one, the induction of magnetic field at some moment of time becomes time-independent, and magnetic field is located near the innermost stable circular orbit. Applications of such systems with nonrotating black holes in real active galactic nuclei are discussed.

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1. Introduction

Black hole accretion flows are the most likely central engine for quasars and active galactic nuclei (AGN) [1-2]. As such they are the subject of intense astrophysical interest and speculation. Recent observations from XMM-Newton, Chandra, Hubble, VLBA, and other ground- and space-based observatories have expanded our understanding of the time variability, spectra, and spatial structure of AGN. Radio interferometry, in particular, has been able to probe within a few hundred gravitational radii (GM/c^2) of the central black hole, e.g. [3-5]. Despite these observational advances, only instruments now in the concept phase will have sufficient angular resolution to spatially resolve the inner accretion disk [6]. And so there remain fundamental questions that we can only answer by folding observations through models of AGN structure.

All black hole accretion flow models require that angular momentum be removed from the flow in some way so that material can flow inwards. In one group of models, angular momentum is removed directly from the inflow by, e.g., a magneto-centrifugal wind [7]. Here we will focus on the other group of models in which angular momentum is diffused outward through the accretion flow.

It has long been suspected that the diffusion of angular momentum through an accretion flow is driven by turbulence. The α model [8] introduced a phenomenological shear stress into the equations of motion to model the effects of this turbulence. This shear stress is proportional

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to αP , where α is a dimensionless constant and P is the (gas or gas + radiation) pressure. This shear stress permits an exchange of angular momentum between neighboring, differentially rotating layers in an accretion disk. In this sense it is analogous to a viscosity [9] and is often referred to as the "anomalous viscosity".

The α model artfully avoids the question of the origin and nature of turbulence in accretion disks. This allows useful estimates to be made absent the solution to a difficult, perhaps intractable, problem. Recently, however, significant progress has been made in understanding the origin of turbulence in accretion flows. It is now known that, in the magnetohydrodynamic (MHD) approximation, an accreting, differentially rotating plasma is destabilized by a weak magnetic field [10-11]. This magneto-rotational instability (MRI) generates angular momentum transport under a broad range of conditions. Numerical work has shown that in a plasma that is fully ionized, which is likely the case for the inner regions of most black hole accretion flows, the magnetorotational instability is capable of sustaining turbulence in the nonlinear regime [11-14].

AGN may be powered by the electromagnetic braking of a rapidly rotating black hole. The Blandford and Znajek [15] effect (here, broadly defined as any electromagnetic means of extracting energy from a rotating black hole) is the most likely astrophysical means of extracting energy from a rapidly rotating black hole. Estimates for the nominal black hole spin in astrophysical environments give a rapid black hole spin of about $a \sim 0.92$ [16]. Phenomenological estimates determined that the Blandford and Znajek luminosity is likely small compared to the disk luminosity [17-19].

Work on magnetized disks has now turned to global numerical models. These are possible thanks to advances in computer hardware and algorithms. Recent work by Hawley, Hawley and Krolik, and Stone and Pringle [13-14,20] considers the evolution of inviscid, nonrelativistic MHD accretion flows in two or three dimensions. Some of this work uses a pseudo-Newtonian, or Paczynski and Wiita [21], potential as a model for the effects of strong-field gravity near the event horizon.

Other work on global models has considered the equations of viscous, compressible fluid dynamics as a model for the accreting plasma [22-25]. The viscosity is meant to mock up the effect of small scale turbulence, presumably generated by magnetic fields, on the large scale flow. In light of work on numerical MHD models, this may seem like a step backwards.

The existing MHD models of accretion disks and flows, however, are computationally expensive, complicated and introduce new problems with respect to initial and boundary conditions. Therefore, there no models today (even numerical), which are able to describe dynamics and evolution of accretion disks in account of time-dependent accretion flow regimes, hydromagnetic dynamo, large spin of a central black hole, and electromagnetic extracting of its energy. In this paper we propose analytical model of nonrotating black hole magnetoshere dynamics, based on solutions of general relativistic MHD equations, taking into account hydromagnetic dynamo effect.

2. Dynamo process in accretion disks

In this section we derive basic equations of GRMHD induction and find their solutions governing dynamo action in accretion disk around nonrotating Schwarzschild black hole. Doing this we are interested mainly in nonstationary solutions, which can show dynamics of accretion disk magnetic fields. We use well known in general relativity formalism where $c = G = 1$.

Let's consider two dimensional field of flowing in matter velocities:

$$\overrightarrow{V} = V_{\varphi}\hat{e}_{\varphi} - V_r\hat{e}_r. \tag{1}$$

Matter in the inner disk regions are mainly in plasma state. Therefore, dynamo action can take place in it. For perfect plasma with infinite conductivity we can write equations of magnetohydrodynamics in general relativity (GRMHD). Following Komissarov, Shibata and Sekiguchi formalism [26-27] we have

$$\begin{cases} \frac{\partial}{\partial t}\left(\sqrt{-g}B^i\right) = -\frac{\partial}{\partial x_j}\left[\sqrt{-g}\left(b^j u^i - b^i u^j\right)\right], \\ \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x_i}\left(\sqrt{-g}B^i\right) = 0, \quad \{i,j\} = (1,2,3), \end{cases} \tag{2}$$

where

$$\begin{cases} b^t = B^i u^\mu g_{i\mu}, \\ b^i = \left(B^i + b^t u^i\right)/u^t, \end{cases} \tag{3}$$

$\mu = (1,2,3,4)$, $B^i = F^{*it}$ — components of magnetic field vector, $u^\mu = \frac{dx^\mu}{ds}$ — 4-velocity of accretion disk matter. In (2) g is determinant of Schwarzschild metric tensor:

$$\det g_{\mu\nu} = -r^4 \sin^2\theta. \tag{4}$$

As accretion disk is assumed to be two dimensional, $\theta = \pi/2$, and $\sqrt{-g} = r^2$. System (2) can be rewritten in the form:

$$\begin{cases} \frac{\partial}{\partial t}\left(\sqrt{-g}B^r\right) = -\frac{\partial}{\partial \phi}\left[\sqrt{-g}\left(b^\phi u^r - b^r u^\phi\right)\right], \\ \frac{\partial}{\partial t}\left(\sqrt{-g}B^\phi\right) = -\frac{\partial}{\partial r}\left[\sqrt{-g}\left(b^r u^\phi - b^\phi u^r\right)\right], \end{cases} \tag{5}$$

and

$$\begin{cases} \frac{\partial}{\partial t}\left(\sqrt{-g}B^r\right) = \\ = -\frac{\partial}{\partial \phi}\left[\sqrt{-g}\left(B^\phi u^r/u^t - B^r u^\phi/u^t\right)\right], \\ \frac{\partial}{\partial t}\left(\sqrt{-g}B^\phi\right) = \\ = -\frac{\partial}{\partial r}\left[\sqrt{-g}\left(B^r u^\phi/u^t - B^\phi u^r/u^t\right)\right]. \end{cases} \tag{6}$$

In consequence of accretion symmetry along direction of ϕ

$$\begin{cases} B^\phi = B^\phi(r,t), \\ B^r = B^r(r,t), \end{cases} \tag{7}$$

we can write:

$$\frac{\partial}{\partial t}(\sqrt{-g}B^r) = 0. \tag{8}$$

The last expression leads to $B^r\sqrt{-g} = \text{const}$. Therefore radial component of magnetic field vector doesn't depend on coordinate time t:

$$B^r = B_0^r\left(\frac{r_0}{r}\right)^2. \tag{9}$$

In (9) r_0 — outer radius of the accretion disk, B_0 — initial value of magnetic field, frozen in interstellar medium in the center of a galaxy. In further calculations B_0 is assumed to be 10^{-7} G [28].

The next our step is to derive analytical expression for B^ϕ. Taking into account (9) together with the second equation of (6) we have:

$$r^2\frac{\partial B^\phi}{\partial t} = -\frac{\partial}{\partial r}\left[B_0 r_0^2\frac{u_\phi}{u_t} - r^2 B^\phi\frac{u_r}{u_t}\right]. \tag{10}$$

As $u_{\phi,r}/u_t = V_{\phi,r}$, its possible to rewrite the last equation:

$$\frac{\partial B^\phi}{\partial t} - \frac{\partial B^\phi}{\partial r}V_r = B^\phi\left(\frac{2}{r}V_r + \frac{\partial V_r}{\partial r}\right) - B_0\left(\frac{r_0}{r}\right)^2\frac{\partial V_\phi}{\partial r}. \tag{11}$$

Derived equation (11) is differential equation with partial derivatives. Its solution gives us B^ϕ as a function of t and r. Its necessary to note that we always use coordinate time (not proper one) through this paper. Equation (11) is equivalent to the system of ordinary differential equations of the first order:

$$dt = -\frac{dr}{V_r} = \frac{dB^\phi}{B^\phi\left(\frac{2}{r}V_r + \frac{\partial V_r}{\partial r}\right) - B_0\left(\frac{r_0}{r}\right)^2\frac{\partial V_\phi}{\partial r}}. \tag{12}$$

The first equation of this system,

$$\frac{dB^\phi}{dt} = B^\phi\left(\frac{2}{r}V_r + \frac{\partial V_r}{\partial r}\right) - B_0\left(\frac{r_0}{r}\right)^2\frac{\partial V_\phi}{\partial r}, \tag{13}$$

has solution

$$B^\phi = B_0\left(\frac{r_0}{r}\right)^2\frac{\partial V_\phi}{\partial r}\frac{1}{2V_r/r + \partial V_r/\partial r} + \\ + C_1\exp\left(2V_r/r + \partial V_r/\partial r\right)t. \tag{14}$$

Taking into account our definition of initial velocity field (1), we should change sign in front of V_r. Therefore $V_r \longrightarrow -V_r$, and

$$B^\phi = -B_0\left(\frac{r_0}{r}\right)^2\frac{\partial V_\phi}{\partial r}\frac{1}{2V_r/r + \partial V_r/\partial r} + \\ + C_1\exp\left(-2V_r/r - \partial V_r/\partial r\right)t. \tag{15}$$

The second equation of the system (12) with its solution looks like

$$dt = \frac{dr}{V_r} \Longrightarrow C_2 = r - V_r t. \tag{16}$$

In the derived expressions C_1 and C_2 are constants which should be found.

The general solution of (12) is arbitrary function

$$F(\Phi_1, \Phi_2) = 0, \tag{17}$$

where

$$\begin{cases} \Phi_1 = C_1(r,t), \\ \Phi_2 = C_2(r,t). \end{cases} \tag{18}$$

We choose linear function for F:

$$F(\Phi_1, \Phi_2) = a_0 + a_1\Phi_1(r,t) + a_2\Phi_2(r,t). \tag{19}$$

We can make one of the coefficient to be equal to unity (e.g. $a_1 := 1$). Therefore we have:

$$B^\phi = -B_0\left(\frac{r_0}{r}\right)^2 \frac{\partial V_\phi}{\partial r}\frac{1}{2V_r/r + \partial V_r/\partial r} + \\ + \left[a_0 + a_2(r - V_r t)\right] e^{-(2V_r/r + \partial V_r/\partial r)t}. \tag{20}$$

During further calculations we normalize (make dimensionless) radial and time coordinate, i.e. $R := r/M$, and $T := t/M$, where M is mass of a central black hole. The next step is to find constants a_0 and a_2 using initial and boundary conditions. At $T = 0$ magnetic field is located at $R = R_0$, and it is equal to B_0, therefore

$$a_0 = B_0 - a_2 R_0. \tag{21}$$

Boundary condition looks like

$$B^\phi\big|_{R=R_0} = B_0. \tag{22}$$

To find explicit form of B^ϕ we need to derive expression for V_r. It's impossible to find it neither theoretically nor experimental today, we use estimation $V_r = \alpha V_\phi$, where α is considerably lesser than unity constant. It is assumed to be equal 10^{-6} during our calculations. Such a case corresponds to rather slow accretion and viscous disk. $V_\phi = 1/\sqrt{R}$ is keplerian velocity of tangent motion. Therefore (22) leads to:

$$a_2 = \frac{\sqrt{R_0}}{\alpha T} B_0 \left[1 + \left(\frac{1}{3\alpha} - 1\right)\exp\left\{\frac{3}{2}\alpha R_0^{-3/2} T\right\}\right]. \tag{23}$$

Combining (21), (23) with (20) we have

$$B^\phi = B_0\left(\frac{R_0}{R}\right)^2 \frac{1}{3\alpha} + B_0\left[\exp\left\{-\frac{3}{2}\alpha R_0^{-3/2} T\right\} - \\ - \frac{R_0\sqrt{R_0}}{3\alpha^2 T} + \frac{\sqrt{R_0}}{3\alpha^2 T}\left(R - \frac{\alpha T}{\sqrt{R}}\right)\right] \times \\ \times \exp\left\{-\frac{3}{2}\alpha T\left(\frac{1}{R^{3/2}} - \frac{1}{R_0^{3/2}}\right)\right\}. \tag{24}$$

But formula (24) isn't the final solution for magnetic field component B^ϕ. It's necessary to take into account the fact that magnetic field exists only in the region where accreted matter have already been arrived, i.e. in the region $R \geq R_0 - \langle V_r \rangle_r T$. More exactly, this condition should be written as

$$\int_R^{R_0} \frac{dR}{V_r} \leq \int_0^T dT \implies R^{3/2} \geq R_0^{3/2} - \frac{3}{2}\alpha T. \tag{25}$$

Expressions (9) and (24) together with condition (25) define distribution of magnetic field vector components along the plane of accretion disk and its time dependence.

3. Discussion and conclusion

Based on results of analytical derivations in the previous section it is possible to plot spatial and temporary distribution of magnetic-field vector in the accretion disk. To do this we need to

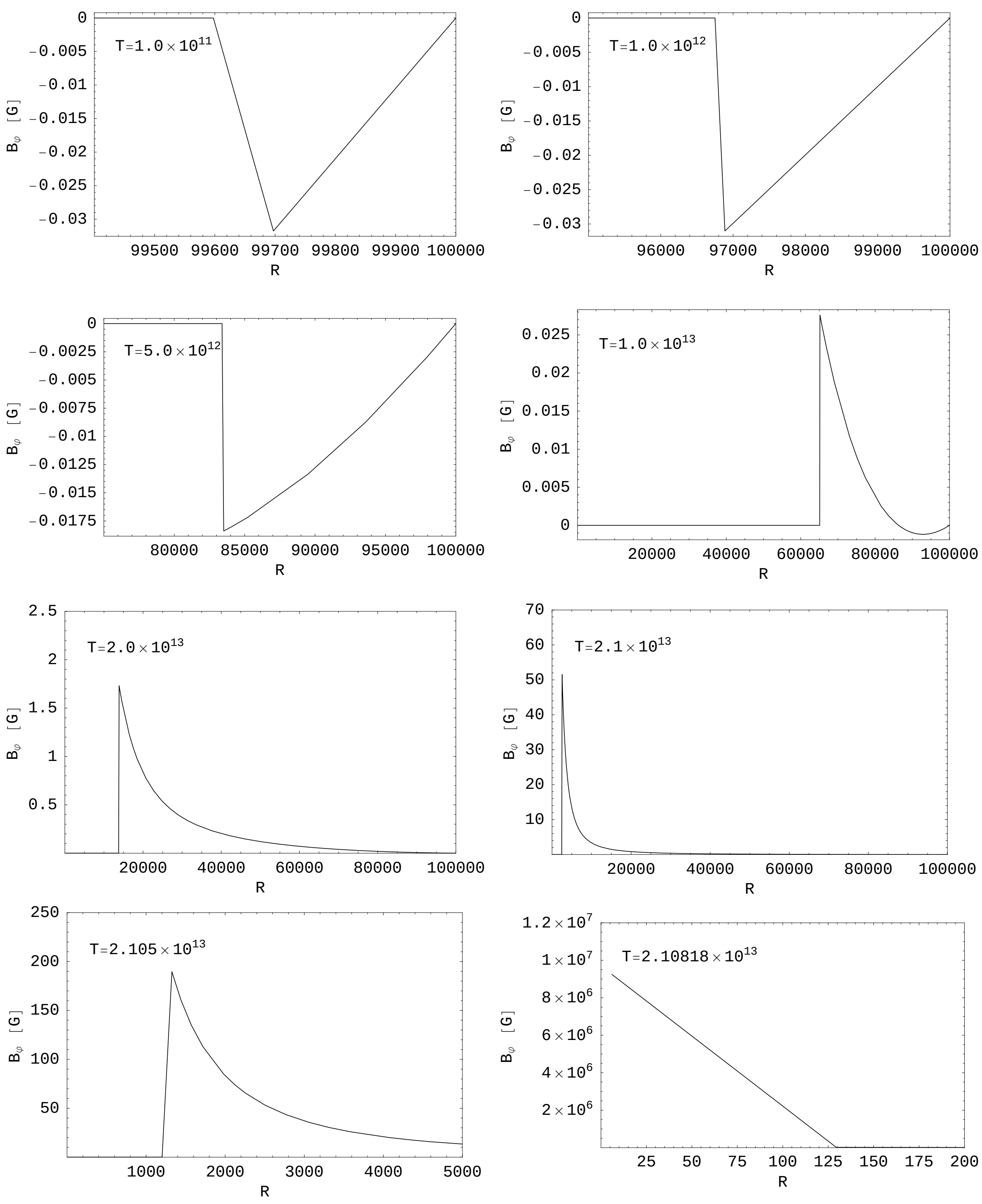

FIG. 1. Distribution of azimuth component of magnetic-field vector along radial coordinate of the accretion disk plane. Plots correspond to moments of time T: 10^{11}, 10^{12}, 5×10^{12}, 10^{13}, 2×10^{13}, 2.1×10^{13}, 2.105×10^{13}, 2.10818×10^{13}.

define some input parameters of the model. First of all, initial magnetic field is assumed to be $B_0 = 10^{-7}$ G, as discussed in the previous section. In our model we need to know exactly the explicit form of the expression for radial velocity in the disk. However, this is linked with great difficulties today [29-31]. Unfortunately problem of viscosity in the disk, which defines radial velocity of accreted plasma, is far from its solution today. Therefore we use the estimation for radial component of plasms velocity $V_r = \alpha V_\phi$, where $V_\phi = R^{-1/2}$ — Keplerian velocity of a circular orbit. The main problem of this model is to estimate dimensionless parameter α, which can be connected with viscosity process in the accretion disk. Our "viscosity parameter" α is different from α-parameter in standard accretion model (Shakura and Sunyaev 1973). We assume our α value to be 10^{-6}. This case corresponds to rather viscous disk and slow accretion. Such a case seems to be common in real astrophysical conditions, and contemporary accretion models (e.g. [13-14,29]) are in a good correspondence with it. The last arbitrary parameter of the model is outer radius of the accretion disk R_0. We assume it to be equal 100 000. This value of R_0 means approximately 1 pc in diameter of the accretion disk for an average supermassive black hole $10^6 - 10^7 M_\odot$.

The result plots are presented in fig.1. As in the previous section distance R and time T are expressed in units of central black hole mass M. Fig. 1 shows several plots with R-distribution of magnetic-field vector $\vec{B}$ ($\vec{B} \simeq B_\phi \hat{e}_\phi$, as $B_\phi \gg B_r$) for different moment of coordinate time T. Is possible to see the "wave of magnetic field" that propagates from the outer borders of the disk R_0 inwards. At the initial moment of time $T = 0$ magnetic field is on the outer border of the disk, where $B = B_0 = 10^{-7}$ G. In the inner parts of the disk $B = 0$ as there is no accreted plasma there. Then nonzero magnetic field is aligned with plasma motion via dynamo action. In fig. 1 it's possible to see different stages of dynamo process. All of them are essentially nonlinear. The most noticeable feature of the process is burst of magnetic field near the time dependent inner border of the accretion disk. In spite of large scales of the dynamo action, for the space of whole accretion disk, induced magnetic field is rather small. It rises greatly (up to $\sim 10^7$ G for $\alpha = 10^{-6}$) only when achieved the radius of innermost stable circular orbit for Schwarzschild black hole ($R = 6$).

Analyzing results obtained during our analytical modeling, it's possible to notice two main stages of accretion and magnetic field induction. They are: transient regime and steady-state condition. Transition between them is realized when accreted plasma reaches the innermost stable orbit, i.e. $R = 6$. Such a transition take place at $T > T_a$, where T_a is period of accretion:

$$T_a = -\int_{R_0}^{6} \frac{dR}{V_r} = 2.10818 \times 10^{13}. \tag{26}$$

At $T > T_a$ steady-state regime is aligned. Accretion and dynamo process become time-independent. To estimate such a value of T_a we come back into SI system. Accretion period in seconds

$$t_a = T_a \frac{GM}{c^3}, \tag{27}$$

and, finally,

$$t_a \text{ [years]} = 3.296 \frac{M}{M_\odot}. \tag{28}$$

Fig 2. illustrates the last formula. It shows dependence between mass of the central supermassive black hole and accretion period. It possible to see that t_a achieves the value of $\sim 3 \times 10^9$ years for $M/M_\odot = 10^9$. This case corresponds to e.g. active galactic nucleus of M87 (Ferrarese and Ford [32]).

The last point to be discussed is the role of Schwarzschild black holes in AGN activity. AGNs seem to contain fast spinning black holes in their centers. This is inevitable phenomena when angular momentum is conserved during black hole formation in the centers of parent galaxies. Schwarzschild black holes could be possible at late stages of AGN evolution when initially Kerr black holes lose their angular momentum and become nonrotating. They also seem to be possible at eventual repeats of AGN activity.

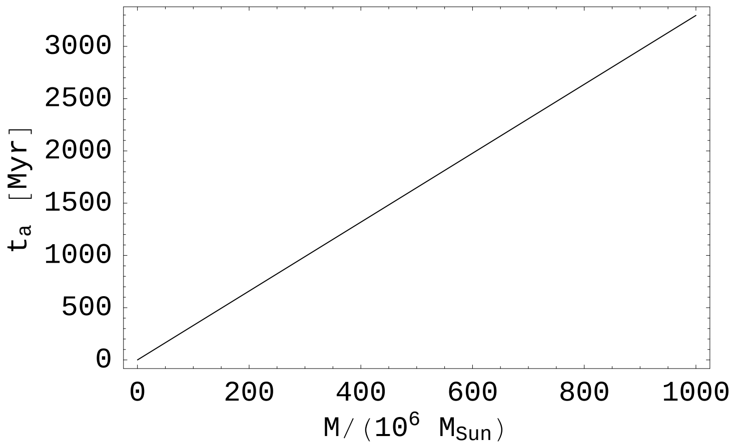

FIG. 2. Mass of the central black hole in AGN — accretion period dependence. Accretion period t_a is expressed in million years, black hole mass M — in million solar masses.

Thus, main results of this paper are following.

- We found analytical solutions of GRMHD equations of dynamo effect in accretion disks of supermassive nonrotating black holes.
- It was shown that radial component of magnetic-field vector B_r is time independent, and $B_\phi \gg B_r$ when $V_\phi \gg V_r$.
- It was found that dynamo action consists of two regimes: transient and steady-state. During steady-state mode magnetic field is located near the inner border of the accretion disk. It reaches up to $\sim 10^7$ G at the innermost stable circular disk orbit.
- Such a value of maximal magnetic field is rather low comparatively to pulsars and magnetars.
- The existence of such systems with nonrotating black holes is possible during late stages of AGN evolution and repeats of their activity.

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