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Phenomenological consequences of the neutrino anomalous magnetic moment

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The models with the massive neutrinos and "see-saw" mechanism are considered. The influence of the neutrino anomalous magnetic moment (AMM) on the value of the muon AMM is investigated. It is shown that the contributions coming from the diagrams with the heavy neutrino exchange could lead to the value of the muon AMM measured in BNL'01 experiment.

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It is well known that a neutrino with non-zero mass has non-trivial electromagnetic properties. In particular, the radiative corrections (RC) generate the dipole magnetic moment (MM) for the Dirac massive neutrino. In the minimally extended standard model (SM) with SU(2)-singlet right-handed neutrino the one-loop RC generate neutrino MM which is proportional to the neutrino mass

$$\mu_\nu = \frac{3eG_F m_\nu}{8\sqrt{2}\pi^2} = 3\times 10^{-19}\mu_0\left(\frac{m_\nu}{1\mathrm{eV}}\right), \tag{1}$$

where $\mu_0 = e/2m_e$ is the Bohr magneton, m_ν (m_e) is the neutrino (electron) mass and the charged lepton mass was set to zero under calculation. There are also models in which much large values for magnetic moments of neutrinos are predicted. One of such examples is the left right model (LRM). This model predicts light and heavy neutrinos in every generation. If one neglects the contributions coming from the charged Higgs bosons then the expression for the MM of the heavy neutrino follows from Eq. (1) under replacement

$$m_\nu \to m_N \sin^2\xi, \tag{2}$$

where ξ is the mixing angle in the sector of the charged gauge bosons. Estimations of the heavy neutrino masses, based on using the data of inverse muon decays and the constraints on the masses of the charged Higgs bosons and W_2 bosons, have been done in Ref. [1]. It was shown that the current lower limit on the heavy neutrino mass reaches 92 GeV. Thus, the constraint on the MM of the heavy neutrino has the form

$$\mu_N \leq 2.7\times 10^{-8}\mu_0. \tag{3}$$

We note that up to now there are no experimental limitations on μ_N. The laboratory constraints exist for light neutrino MMs only. At present the more stringent limitations on them come from elastic neutrino-electron scattering experiments and are given by the expresions [2]

$$\mu_{\nu_e} \leq 1.5\times 10^{-10}\mu_0, \qquad \mu_{\nu_\mu} \leq 6.8\times 10^{-10}\mu_0, \qquad \mu_{\nu_\tau} \leq 3.9\times 10^{-10}\mu_0. \tag{4}$$

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The goal of our work is to define the phenomenological consequences of the neutrino anomalous magnetic moment (AMM). We shall carry out our investigation within the LRM. However, the suggested scheme could be used for any gauge electroweak theory with massive neutrinos and the "sea-saw" mechanism.

We start with considering the influence of the neutrino AMM on the value of the muon AMM. We recall that the muon MM is related to its intrinsic spin by the gyromagnetic ratio g_μ:

$$\boldsymbol{\mu} = g_\mu \left(\frac{q}{2m}\right)\mathbf{S},$$

where $g_\mu = 2$ is expected for structureless, spin 1/2 particle of mass m and charge $q = \pm e$. The RC, which couple the muon spin to virtual fields, introduce an AMM defined by

$$a_\mu = \frac{1}{2}(g_\mu - 2).$$

The leading RC is the lowest-order (LO) quantum electrodynamic process involving the exchange of a virtual photon, the "Schwinger term" giving $a_\mu(QED; LO) = \alpha/2\pi \approx 1.16 \times 10^{-3}$. The complete SM value of a_μ, currently evaluated to a precision of approximately 0.6 ppm, includes this first-order term along with higher order QED processes, electroweak loops, hadronic vacuum polarization, and other higher order hadronic loops. The difference between experimental and theoretical values for a_μ is a valuable test of the completeness of the SM. At sub-ppm precision, such a test explores physics well above the 100 GeV scale for many SM extensions. The muon AMM was measured in a series of three experiments at CERN (during the years 1968-1977) and, most recently in E821 experiment at Brookhaven National Laboratory (BNL) on Alternating Gradient Synchrotron. The last CERN-experiment (CERN-III) used a uniform-field storage ring and electric quadrupoles to provide vertical containment for the muons having the "magic" momentum of 3.1 GeV/c. At this momentum, the muon spin precession is not affected by the electric field from the focusing quadrupoles. The the results of the CERN-III experiment were combined to give a 7.3 ppm measurement, which agreed with theory. The present BNL experiment follows the general technique pioneered by CERN-III. Data obtained at BNL, using nearly equal samples of positive and negative muons, were used to deduce $a_\mu^{exp} = 11659208.0(5.4)(3.3) \times 10^{-10}$, where the statistical and systematic uncertainties are given, respectively. The combined uncertainty of 0.54 ppm represents a 14-fold improvement compared to previous measurements at CERN.

In order for theory to match such an accurate measurement, calculations in the SM have to be pushed to their very limits. So, to compare with the experimental accuracy prediction the contributions of all sectors of the SM (or its extension) have to be known very precisely. Therefore, the SM expression for the muon AMM a_μ^{SM} should include the following terms

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}, \tag{5}$$

in which $a_\mu^{QED} = 11\ 658\ 4705.7(2.9) \times 10^{-11}\mu_0$ (see [3] and references therein). The electroweak contribution from one and two loops are

$$a_\mu^{EW} = 15.4(0.1)(0.2) \times 10^{-10},$$

where the first error comes from two-loops ekectroweak hadronic effects in the quarl triangle diagrams and the second comes from the uncertainty on the Higgs boson mass [4]. The term a_μ^{had} is mainly defined by virtual hadronic contributions to the photon propagator in 4^{th} $a_\mu^{had}(VP1)$ and 6^{th} order, where the latter includes hadronic vacuum polarization $a_\mu^{had}(VP2)$ and light-by-light scattering $a_\mu^{had}(LbyL)$. The hadronic contributions on the level of Feynman diagrams arise through loops of virtual quarks and gluons. These loops also involve the soft scales, and therefore cannot be computed reliably in perturbative QCD. The analysis made in Ref. [5] showed a discrepancy between the $a_\mu^{had}(VP1)$ value obtained exclusively from e^+e^- data and that which arises if only τ data are used. We recall that the inclusion of τ-decay data introduces systematic uncertainties originating from isospin symmetry breaking effects which are difficult to estimate. For this reason some authors are not including τ data and are working with the e^+e^- data only. At present two SM results for a_μ are used

$$a_\mu^{SM} = 11659185.7(8.0) \times 10^{-10}, \qquad [6], \tag{6}$$

$$a_\mu^{SM} = 11659182.0(7.3) \times 10^{-10}, \qquad [7]. \tag{7}$$

These results represent the two slightly different e^+e^- based evaluations of the leading-order hadronic vacuum polarization contribution. In the obtained value for a_μ the largest theoretical uncertainty, 0.55 ppm, is associated with first-order hadronic vacuum polarization. Introducing the quantity

$$\delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM},$$

we obtain

$$\frac{\delta a_\mu}{\mu_0} = (22.4 \pm 10) \times 10^{-10}, \tag{8}$$

$$\frac{\delta a_\mu}{\mu_0} = (26.1 \pm 9.4) \times 10^{-10}. \tag{9}$$

So, these SM evaluations, based on e^+e^- hadronic cross sections, display 2.2 and 2.7 standard deviations below the experimental result. Further we believe that there is some room for new physics in the BNL-E-0821 results, i.e. the deviation of δa_μ is attributed to effects of the physics beyond the SM. We shall consider the $(g-2)_\mu$ anomaly within the LRM assuming that

$$(a_\mu^{QED} + a_\mu^{had})_{SM} = (a_\mu^{QED} + a_\mu^{had})_{LRM}, \qquad \text{but} \qquad (a_\mu^{EW})_{SM} \neq (a_\mu^{EW})_{LRM}.$$

In Ref. [8] it was shown that the observed value of the muon AMM could be explained by the contributions coming from the Higgs sector of the LRM. Here we draw our attention to the influence of the neutrino AMM on the AMM.

If the difference between the experiment and SM prediction is 2.2 and 2.7 σ, then at 90% CL, $\delta a_\mu/\mu_0$ must lie in the range

$$9.61 \times 10^{-10} \leq \frac{\delta a_\mu}{\mu_0} \leq 35.19 \times 10^{-10}, \tag{10}$$

and

$$14.08 \times 10^{-10} \leq \frac{\delta a_\mu}{\mu_0} \leq 38.12 \times 10^{-10} \tag{11}$$

respectively.

In the LRM neutrino could have both Majorana and Dirac nature. We consider the case of the Dirac neutrino. For the sake of simplicity we assume that the mixing takes plase between the muon and the tau lepton neutrino only. The neutrino AMM induced by the RC leads to the appearance of the following terms in the effective interaction Lagrangian

$$H_{add} = \sum_l \left[\mu_{\nu_l}\overline{\psi}_{\nu_l}(p)\sigma_{\lambda\tau}q_\tau\psi_{\nu_l}(p') + \mu_{N_l}\overline{\psi}_{N_l}(p)\sigma_{\lambda\tau}q_\tau\psi_{N_l}(p')\right] A_\lambda(q), \tag{12}$$

where

$$\sigma_{\lambda\tau} = \frac{i}{2}\left(\gamma_\tau\gamma_\lambda - \gamma_\lambda\gamma_\tau\right).$$

The vertex function of the third order $\Lambda_\beta(p,p')$ which correspond to the diagrams with the W_1^--boson exchange has the form

$$\Lambda_\beta(p,p') = \frac{\mu g^2}{(4\pi)^4 m_e}\int\left\{c_{\varphi_\mu}^2 c_{\theta_\nu}^2 \frac{\gamma_\sigma(1+\gamma_5)[i(\hat{p}'-\hat{k}) - m_{\nu_2}]\sigma_{\beta\tau}q_\tau[i(\hat{p}-\hat{k}) - m_{\nu_2}]\gamma_\nu(1+\gamma_5)}{[(p'-k)^2 + m_{\nu_2}^2][(p-k)^2 + m_{\nu_2}^2]} + \right.$$

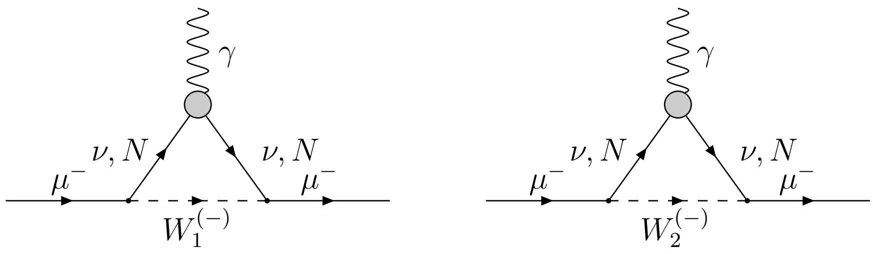

FIG. 1: One-loop diagrams contributing to the muon AMM due to the light and heavy neutrinos.

$$+ (\varphi_\mu \to -\varphi_\mu, \theta_\nu \to \theta_N, m_{\nu_2} \to m_{N_2}) + \left(\varphi_\mu \to \varphi_\mu, \theta_\nu \to \theta_\nu + \frac{\pi}{2}, m_{\nu_2} \to m_{\nu_3}\right) +$$
$$+ \left(\varphi_\mu \to \varphi_\mu + \frac{\pi}{2}, \theta_\nu \to \theta_N + \frac{\pi}{2}, m_{\nu_2} \to m_{N_3}\right)\bigg\} \frac{\delta_{\sigma\nu} + k_\sigma k_\nu / m_{W_1}^2}{k^2 + m_{W_1}^2} d^4k, \qquad (13)$$

where we have took into account the relation

$$\nu_\mu(x) = c_{\varphi_\mu} c_{\theta_\nu} \nu_2(x) - s_{\varphi_\mu} c_{\theta_N} N_2(x) - c_{\varphi_\mu} s_{\theta_\nu} \nu_3(x) + s_{\varphi_\mu} s_{\theta_N} N_3(x), \qquad (14)$$

φ_l is the mixing angle between the light and heavy neutrino in l-generation, θ_ν (θ_N) is the mixing angle between light (heavy) neutrinos belonging to different generations, $s_{\varphi_\mu} = \sin\varphi_\mu, c_{\varphi_\mu} = \cos\varphi_\mu$ and so on.

Of course, since spontaneously broken gauge theories are renormalizable, it is clear that the matrix elements corresponding to the diagrams of Fig.1 should be finite. In fact the total bare Lagrangian does not contain an interaction of the form of the magnetic terms. However, there is an ambiguity in the finite parts of the graphs when computing in the unitary gauge. In that gauge, the propagator of the W_1-boson contains $k_\sigma k_\nu / m_{W_1}^2$ term, giving rise to a linearly divergent piece. Although these divergent terms cancel (to be exact they are absorbed by renormalization), the finite parts depend on the routing of the momenta through the graphs. Such an ambiguity could be resolved in several ways. For example, one could use the procedure of Ref. [9], where the W_1-boson line does not carry any external momenta, and use the ξ-limiting procedure which makes the replacement

$$\frac{k_\sigma k_\nu}{m_{W_1}^2} \frac{1}{k^2 + m_{W_1}^2} \to \frac{k_\sigma k_\nu}{m_{W_1}^2} \frac{m_{W_1}^2 + M^2}{(k^2 + m_{W_1}^2)(k^2 + M^2)},$$

where the limit $M \to \infty$ being taken at the end of the calculation. The second way is to compute the matrix elements in the R_ξ gauge. In that gauge, the propagator of the W_1-boson is given by the expression

$$-i\left[\delta_{\sigma\nu} + \frac{k_\nu k_\sigma}{k^2 + m_{W_1}^2/\xi}\left(1 - \frac{1}{\xi}\right)\right] \frac{1}{k^2 + m_{W_1}^2},$$

and there are also contributions to the graphs coming from would-be Goldstone bosons with mass depending inversely on ξ. Here we choose more simple way, namely, we compute directly in the unitary gauge ($\xi \to 0$) where the particle content is evident, and use the procedure of Dyson [10] based on the expansion in external momenta of the subintegral expression and subsequent subtraction of the divergent terms.

First we define the contribution to the muon AMM coming from the left-handed currents. In doing so we are constrained by the diagonal elements of the neutrino AMM. Straightforward but tedious calculation lead to the result

$$\frac{\delta a_\mu^{W_L}}{\mu_0} = \frac{G_F m_{W_1}^2}{2\sqrt{2}\pi^2 m_e} \bigg\{ c_{\varphi_\mu}^2 c_{\theta_\nu}^2 \cos^2\xi m_{\nu_2} \mu_{\nu_2\nu_2} \bigg[\int_0^1 \bigg(\frac{x(1-x)^2 - x^3 m_{\nu_2}^2 m_{W_1}^{-2}}{(x - m_{W_1}^2 m_\mu^{-2})(x-1) + x m_{\nu_2}^2 m_\mu^{-2}} +$$

$$+m_\mu^2 m_{W_1}^{-2} x(1-3x)\ln\left|\frac{m_{W_1}^2(1-x)+m_{\nu_2}^2 x}{(m_{W_1}^2 - x m_\mu^2)(1-x)+m_{\nu_2}^2 x}\right|\Bigg) dx + \frac{m_\mu^2}{3m_{W_1}^2}\Bigg] +$$
$$+ (\varphi_\mu \to -\varphi_\mu, \theta_\nu \to \theta_N, \mu_{\nu_2\nu_2} \to \mu_{N_2N_2}, m_{\nu_2} \to m_{N_2}) + \left(\varphi_\mu \to \varphi_\mu, \theta_\nu \to \theta_\nu + \frac{\pi}{2}, m_{\nu_2} \to m_{\nu_3}\right) +$$
$$+ \left(\varphi_\mu \to \varphi_\mu + \frac{\pi}{2}, \theta_\nu \to \theta_N + \frac{\pi}{2}, m_{\nu_2} \to m_{N_3}\right) +$$
$$+ c_{\varphi_\mu}^2 c_{\theta_\nu}^2 \sin^2\xi m_{\nu_2}\Bigg[\int_0^1 \Bigg(\frac{x(1-x)^2 - x^3 m_{\nu_2}^2 m_{W_2}^{-2}}{(x - m_{W_2}^2 m_\mu^{-2})(x-1) + x m_{\nu_2}^2 m_\mu^{-2}} +$$
$$+m_\mu^2 m_{W_2}^{-2} x(1-3x)\ln\left|\frac{m_{W_2}^2(1-x)+m_{\nu_2}^2 x}{(m_{W_2}^2 - x m_\mu^2)(1-x)+m_{\nu_2}^2 x}\right|\Bigg) dx + \frac{m_\mu^2}{3m_{W_2}^2}\Bigg] +$$
$$+ (\varphi_\mu \to -\varphi_\mu, \theta_\nu \to \theta_N, m_{\nu_2} \to m_{N_2}) + \left(\varphi_\mu \to \varphi_\mu, \theta_\nu \to \theta_\nu + \frac{\pi}{2}, m_{\nu_2} \to m_{\nu_3}\right) +$$
$$+ \left(\varphi_\mu \to \varphi_\mu + \frac{\pi}{2}, \theta_\nu \to \theta_N + \frac{\pi}{2}, m_{\nu_2} \to m_{N_3}\right)\Bigg\}. \qquad (15)$$

Now we difine the contribution caused by the right handed currents. Since the wave function of the heavy muon neutrino is connected with the physical neutrino states by the relation

$$N_\mu = s_{\varphi_\mu} c_{\theta_\nu} \nu_2(x) + c_{\varphi_\mu} c_{\theta_N} N_2(x) + s_{\varphi_\mu} s_{\theta_\nu} \nu_3(x) + c_{\varphi_\mu} s_{\theta_N} N_3(x), \qquad (16)$$

then to obtain the contribution in question one should fulfill the replacement

$$\xi \to \xi + \frac{\pi}{2}, \qquad \varphi_\mu \to \varphi_\mu + \frac{\pi}{2} \qquad (17)$$

in the expression standing in the braces of Eq. (15).

Taking into account an experimental data ranges for $\delta a_\mu/\mu_0$ (10) the next bounds for AMM of heavy neutrino, due to diagrams with exchange by W_1^- −and W_2^- −bosons, were respectively obtained:

$$9.61 \times 10^{-10} \le \frac{\delta a_\mu}{\mu_0} \le 35.19 \times 10^{-10},$$
$$m_N = 100\text{GeV},\ 9.6 \times 10^{-8} \le \mu \le 3.5 \times 10^{-7};$$
$$m_N = 1000\text{GeV},\ 4.0 \times 10^{-7} \le \mu \le 1.5 \times 10^{-6},$$

for W_1^--diagram and

$$m_N = 100\text{GeV},\ 2.8 \times 10^{-14} \le \mu \le 1.0 \times 10^{-13};$$
$$m_N = 1000\text{GeV},\ 9.6 \times 10^{-15} \le \mu \le 3.5 \times 10^{-14},$$

for W_2^--diagram.

Thus we have investigated the influence of the neutrino AMM on the value of the muon AMM within the models with the massive neutrinos and the "see-saw" mechanism. We have shown that the BNL'01 result could be explained by the heavy neutrino sector contributions. Comparing the theoretical and experimental results we have established the allowed values regions for the neutrino sector parameters. In conclusion we note that although experiment and theory have now both reached the same level of accuracy the present discrepancy between the e^+e^- and τ based evaluations makes the interpretation of the BNL'01 result a delicate issue as far as evidence for new physics signal. It is clear that the prospects for additional high statistics data in the future, either from KLOE or from BaBar, are most welcome. On the other hand, if the present discrepancy in the evaluations of $a_\mu^{had}(VP1)$ finds a solution in the future, and if the experimental error is further reduced, then the theoretical uncertainty on the light-by-light scattering contribution will constitute the next serious limitation for the $(g-2)_\mu$ problem.

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