Nonlinear Dynamics and Applications. Vol. 13 (2006) 133 - 139

Incoherent solitons in two-component amplifying-absorbing dense media

A.M. Lemeza* and R.A. Vlasov†
B.I.Stepanov Institute of Physics of the National Academy of Science of Belarus
68 Nezavisimosti Ave., 220072, Minsk, Belarus

The formation and stability of soliton-like pulses in a dense two-component (amplifying-absorbing) medium is considered. The existence of incoherent solitons in such media is shown, the soliton solutions being unstable with respect to small perturbations.

PACS numbers: 46.65.Tg
Keywords: incoherent solitons, dense media, near dipole-dipole interactions

1. Introduction

Originally the concept of an optical soliton was introduced in studies of propogation of pulses with their unchangeable parameters in conservative media (without absorbtion or amplification of energy). These solitons are steady-state pulses that are formed under certain conditions. Namely, their formation is related to the competition and balance of some factors. In particular, in Kerr-type media, this is the competition between diffraction or dispersion and cubic nonlinearity. In the case of self-induced transparency (SIT) the competitive factors are coherent absorption and stimulated emission [1].

Another kind of stationary (steady-state) wave packets (solitons) is associated with the steady-state pulse propagation in amplifying-absorbing media. These solitons are called dissipative ones. Besides the competition of above mentioned factors, there must be the balance between the amplification and losses. Just this kind of solitons appears in the case of laser generation under the mode-locking conditions. The passive mode-locking implies the presence of two types of media in a resonator (amplifier and absorber). The absorber is responsible for the formation of the shape of a pulse, while the amplifier compensates loss of energy of a pulse.

Generation of dissipative solitons is quite interesting from the practical point of view because of their properties. Namely, their parameters (amplitude and duration) possess maximum stability. For that reason any new soliton-type solution merits notice and is of grate interest for possible applications.

In this report the results of our work are presented concerning laser generation of dissipative optical solitons in two-component media. These media are assumed to be dense to such a degree that it necessary to allow for the so-called local-field effects associated with the near dipole-dipole (NDD) interactions [2]. This differentiates our approach from others. The problem of existence of the incoherent solitons in dense single-component media was studied in [3, 4]. The authors of these papers have shown the necessity of finding the optimal ratio between the detuning from resonant and Lorenz frequency. However the estimated value of the near dipole-dipole interaction parameter is large enough ($b \sim 10$) and this hampers the formation of incoherent solitons in such media.

Below we will demonstrate that the formation of incoherent solitons is possible in two-component dense media for suitable near dipole-dipole interaction parameters. Furthermore we will show that the obtained soliton solution is in general unstable with respect to small perturbations, but the instability growth rate can have a quite low value.

*E-mail: azemel@gmail.com
†E-mail: r.vlasov@dragon.bas-net.by

2. Model

To describe propagation of an electromagnetic field we use Maxwell equation and to describe interaction of a wave with the media we use two Bloch subsystems for each medium. The reduced system of equations for the complex amplitudes of electromagnetic field $\mathcal{E}$ and polarizations $\mathcal{P}_1$ and $\mathcal{P}_2$ can be represented as

$$\frac{\partial \mathcal{E}}{\partial z} + \frac{1}{v_0}\frac{\partial \mathcal{E}}{\partial t} + l_{cav}\mathcal{E} = \frac{i2\pi k_0}{n_0^2}\left(\mu_1 N_1 \mathcal{P}_1 + \mu_2 N_2 \mathcal{P}_2\right), \tag{1}$$

$$\frac{\partial \mathcal{P}_1}{\partial t} = i\frac{\mu_1}{\hbar}\mathcal{E}\mathcal{W}_1 + i\left(\Omega_1 + \Omega_L^{(1)}\mathcal{W}_1\right)\mathcal{P}_1 - \frac{\mathcal{P}_1}{T_2^{(1)}}, \tag{2}$$

$$\frac{\partial \mathcal{W}_1}{\partial t} = i\frac{2\mu_1}{\hbar}\left(\mathcal{E}^*\mathcal{P}_1 - \mathcal{E}\mathcal{P}_1^*\right), \tag{3}$$

$$\frac{\partial \mathcal{P}_2}{\partial t} = i\frac{\mu_2}{\hbar}\mathcal{E}\mathcal{W}_2 + i\left(\Omega_2 + \Omega_L^{(2)}\mathcal{W}_2\right)\mathcal{P}_2 - \frac{\mathcal{P}_2}{T_2^{(2)}}, \tag{4}$$

$$\frac{\partial \mathcal{W}_2}{\partial t} = i\frac{2\mu_2}{\hbar}\left(\mathcal{E}^*\mathcal{P}_2 - \mathcal{E}\mathcal{P}_2^*\right), \tag{5}$$

with the initial conditions

$$\mathcal{P}_{1,2}\left(-\infty\right) = 0, \qquad \mathcal{W}_{1,2}\left(-\infty\right) = \mathcal{W}_{01,02}, \tag{6}$$

where the real electromagnetic field and polarization a taken to be

$$\hat{\mathcal{E}} = \mathcal{E}\exp\left[-i(\omega t - k_0 z)\right] + c.c., \tag{7}$$

$$\hat{\mathcal{P}}_i = \mathcal{P}_i\exp\left[-i(\omega t - k_0 z)\right] + c.c., \quad i = 1,2. \tag{8}$$

Indexes 1 and 2 denote amplifier and absorber respectively, $\mathcal{W}_{1,2}$ and $\mu_{1,2}$ are the population inversions and dipole moments of a resonant transition, $N_{1,2}$ is the concentrations of active and passive atoms, $k_0 = \omega n_0/c$ is the wave vector of the electromagnetic field, $v_0 = c/n_0$ phase velocity of light in the medium, n_0 nonresonant part of the refractive index of the medium, $\Omega_{1,2} = \omega - \omega_{1,2}$ are offsets of frequency ω relative to the centers of amplifying and absorption lines, $\Omega_L^{(1,2)} = 4\pi\mu_{1,2}^2 N_{1,2}/3\hbar$ are the Lorentz frequency determining the frequency shift as a result of the short range dipole-dipole interaction, $T_2^{(1,2)}$ are the transverse relaxation times, l_{cav} is the linear loss, $\hbar$ is the Planks' constant.

We assume that the longitudinal relaxation times $T_1^{(1,2)}$ are much greater than the pulse duration τ ($\tau \ll T_1^{(1,2)}$). In other words, we neglect the relaxation terms in equations (3) and (5).

For the further consideration, it is convenient to represent the equations (1)-(5) in the form

$$\frac{\partial E}{\partial z'} + \alpha\frac{\partial E}{\partial u} + \sigma E = i(\mathcal{P}_1 + \rho\mathcal{P}_2), \tag{9}$$

$$\frac{\partial \mathcal{P}_1}{\partial u} = iE\mathcal{W}_1 - \left(1 - i\left(\Delta_1 + b_1\mathcal{W}_1\right)\right)\frac{\mathcal{P}_1}{T}, \tag{10}$$

$$\frac{\partial \mathcal{W}_1}{\partial u} = 2i\left(E^*\mathcal{P}_1 - E\mathcal{P}_1^*\right), \tag{11}$$

$$\frac{\partial \mathcal{P}_2}{\partial u} = isE\mathcal{W}_2 - \left(1 - i\left(\Delta_2 + b_2\mathcal{W}_2\right)\right)\frac{\mathcal{P}_2}{\kappa T}, \tag{12}$$

$$\frac{\partial \mathcal{W}_2}{\partial u} = 2is\,(E^*\mathcal{P}_2 - E\mathcal{P}_2^*), \tag{13}$$

where

$$\mathcal{E} = \frac{\hbar}{\mu_1\tau}E, \qquad u = \left(t - \frac{z}{v}\right)/\tau, \qquad z' = z\frac{2\pi k_0\mu_1^2 N_1\tau}{n_0^2\hbar},$$
$$\sigma = l_{cav}\frac{n_0^2\hbar}{2\pi k_0\tau\mu_1^2 N_1}, \qquad \alpha = \left(\frac{1}{v_0} - \frac{1}{v}\right)\frac{n_0^2\hbar}{2\pi k_0\mu_1^2 N_1\tau^2}, \qquad \rho = \frac{\mu_2 N_2}{\mu_1 N_1}, \tag{14}$$
$$s = \frac{\mu_2}{\mu_1}, \quad T = \frac{T_2^{(1)}}{\tau}, \quad \kappa T = \frac{T_2^{(2)}}{\tau}, \qquad \Delta_{1,2} = \Omega_{1,2}T_2^{(1,2)}, \quad b_{1,2} = \Omega_L^{(1,2)}T_2^{(1,2)}.$$

In what follows we shall use an approximate approach to solve system (9)-(13).

3. Approximate solutions of the Bloch subsystems

In this section we consider an approximate solution of the Bloch subsystem. We assume that the propagation of a light pulse satisfying the condition $T_2^{(1,2)} \ll \tau$. This means that the polarization of active and passive atoms ($\mathcal{P}_1$ and $\mathcal{P}_2$) follow the change of the pulse field and we can assume in (2) and (4) that $|\partial\mathcal{P}_{1,2}/\partial t| \ll |\mathcal{P}_{1,2}/T_2^{(1,2)}|$.

Here we in detail consider approximate solution of Bloch subsystem (10)-(13) which describes the behaviour of passive atoms. And then we will easily obtain the approximate solution of subsystem (10)-(11) by the corresponding replacement.

At first we assume that $\partial\mathcal{P}_2/\partial u = 0$. Then from equation (12) we obtain expression for the polarization

$$\mathcal{P}_2 = \frac{i\kappa T s E\mathcal{W}_2}{1 - i\,(\Delta_2 + b_2\mathcal{W}_2)}. \tag{15}$$

Then equation (13) can be transformed to the form

$$\frac{\partial \mathcal{W}_2}{\partial u} + \frac{4s^2\kappa T\mathcal{W}_2|E|^2}{1 + (\Delta_2 + b_2\mathcal{W}_2)^2} = 0. \tag{16}$$

Under initial condition (6) the obtained equation, on separating the variables, can be integrated to yield

$$\begin{gathered}\left(\left(\mathcal{W}_2^2 - \mathcal{W}_{02}^2\right)b_2\right)^2 + 4\,(\mathcal{W}_2 - \mathcal{W}_{02})\,b_2\Delta_2 + \\ +2\,(\ln(\mathcal{W}_2) - \ln(\mathcal{W}_{02}))\left(1 + \Delta_2^2\right) + 8s^2\kappa T\int\limits_{-\infty}^{u}|E|^2\,du = 0.\end{gathered} \tag{17}$$

Next, it is assumed that the difference of populations changes negligibly during passage of the pulse, i.e., $\mathcal{W}_2 = \mathcal{W}_{02} + w_2$, $w_2 \ll \mathcal{W}_{02}$. Then equation (17) reduces to

$$\mathcal{W}_2 = \mathcal{W}_{02} - w_2 = \mathcal{W}_{02}\left(1 - \frac{4s^2\kappa T}{|\gamma_2|^2}\int\limits_{-\infty}^{u}|E|^2\,du\right), \tag{18}$$

where $\gamma_2 = 1 - i\,(\Delta_2 + b_2\mathcal{W}_{02})$.

Accordingly, the polarization amplitude (15) is now determined by the expression

$$i\mathcal{P}_2 = -E\left((\chi_5 + i\chi_6) + (\chi_7 + i\chi_8)\int\limits_{-\infty}^{u} |E|^2\, du\right), \tag{19}$$

where

$$\chi_5 = \frac{s\kappa T\mathcal{W}_{02}}{|\gamma_2|^2}, \quad \chi_6 = \frac{s\kappa T\mathcal{W}_{02}\left(\Delta_2 + b_2\mathcal{W}_{02}\right)}{|\gamma_2|^2}, \quad \chi_7 = \frac{4s^3\kappa^2T^2\mathcal{W}_{02}\left(1 + \Delta_2^2 - b_2^2\mathcal{W}_{02}^2\right)}{|\gamma_2|^6},$$
$$\chi_8 = \frac{4s^3\kappa^2T^2\mathcal{W}_{02}\left(b_2\mathcal{W}_{02} + \left(\Delta_2 + b_2\mathcal{W}_{02}\right)\left(\Delta_2 b_2\mathcal{W}_{02} + \left(1 + \Delta_2^2\right)\right)\right)}{|\gamma_2|^6}. \tag{20}$$

Now we can readily obtain the approximate solution of Bloch subsystem (10)-(11). To get the expression for the polarization, it is enough to substitute index 2 for 1 in equations (19)-(20). Taking $\kappa \to 1$ and $s \to 1$, we obtain the expression for the polarization of the amplifier:

$$i\mathcal{P}_1 = -E\left((\chi_1 + i\chi_2) + (\chi_3 + i\chi_4)\int\limits_{-\infty}^{u} |E|^2\, du\right), \tag{21}$$

where $\gamma_1 = 1 - i\left(\Delta_1 + b_1\mathcal{W}_{01}\right)$,

$$\chi_1 = \frac{T\mathcal{W}_{01}}{|\gamma_1|^2}, \quad \chi_2 = \frac{T\mathcal{W}_{01}\left(\Delta_1 + b_1\mathcal{W}_{01}\right)}{|\gamma_1|^2}, \quad \chi_3 = \frac{4T^2\mathcal{W}_{01}\left(1 + \Delta_1^2 - b_1^2\mathcal{W}_{01}^2\right)}{|\gamma_1|^6},$$
$$\chi_4 = \frac{4T^2\mathcal{W}_{01}\left(b_1\mathcal{W}_{01} + \left(\Delta_1 + b_1\mathcal{W}_{01}\right)\left(\Delta_1 b_1\mathcal{W}_{01} + \left(1 + \Delta_1^2\right)\right)\right)}{|\gamma_1|^6}. \tag{22}$$

Substituting polarizations $\mathcal{P}_1$ and $\mathcal{P}_2$ into (9), we obtain the final equation which describes the propagation of a pulse in the two-component medium

$$\frac{\partial E}{\partial z'} + \alpha\frac{\partial E}{\partial u} + E\left((\xi_1 + i\xi_2) + (\xi_3 + i\xi_4)\int\limits_{-\infty}^{u} |E|^2\, du\right) = 0, \tag{23}$$

where

$$\xi_1 = \chi_1 + \rho\chi_5 + \sigma, \qquad \xi_j = \chi_j + \rho\chi_{j+4}, \qquad j = 2..4 \tag{24}$$

It is obvious that in such a way we can consider the case of multicomponent media with an arbitrary number of types of active and passive atoms.

4. Incoherent soliton solution

Following the standard procedure, we represent the amplitude E of the propagating pulse in the form

$$E(u) = A(u)\exp\left(i\varphi(u)\right) \tag{25}$$

Substituting equation (25) in to (23), we obtain two real equations for the envelop $A(u)$ and phase $\varphi(u)$

$$\alpha\frac{\partial A(u)}{\partial u}+A(u)\left(\xi_1+\xi_3\int\limits_{-\infty}^{u}A(u)^2\,du\right)=0,$$
$$\alpha\frac{\partial \varphi(u)}{\partial u}+\left(\xi_2+\xi_4\int\limits_{-\infty}^{u}A(u)^2\,du\right)=0. \tag{26}$$

The solution of the system of the equations (26) is

$$A=\frac{A_0}{\cosh{(u)}},\quad \varphi=\frac{1}{\alpha}\left(\xi_2 u+\xi_4 A_0^2\left(u+\ln\left(\cosh\left(u\right)\right)\right)\right),\quad A_0^2=-\frac{\xi_1}{\xi_3}>0,\quad \alpha=-\xi_1. \tag{27}$$

Let us consider the obtained solution. The parameter α is determined by the inverse velocities of light and soliton. If it is less than zero, the the phase velocity of light is greater than the velocity of the pulse and vice versa. The dependence of the normalized inverse velocity of the pulse is shown in Fig.1.

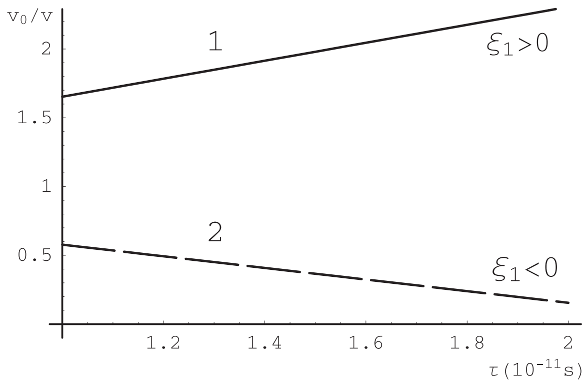

FIG. 1. The evolution of the inverse normalized velocity of pulse versus its duration for decreasing (1) and increasing (2) velocity of the pulse (when $\omega=10^{14}s^{-1}$, $T_2^{(1)}=T_2^{(2)}=10^{-10}s$, $l_{cav}=4\cdot10^{-4}cm^{-1}$, $N_1=10^{19}cm^{-3}$, $N_2=4.1\cdot10^{19}cm^{-3}$, $\mu_1=2\cdot10^{-19}$ CGSE, $\mu_2=10^{19}$ CGSE, $\Omega_1=\Omega_2=10^{-7}s^{-1}$, $n_0=1.2$, $\mathcal{W}_{01}=-1$, $\mathcal{W}_{02}=1$ and $\mathcal{W}_{02}=0.96$) for (1) and (2) correspondingly .

For the cubic approximation (18) to be valid, the following inequalities should be satisfied:

$$\frac{8A_0^2T}{|\gamma_1|^2}\ll1,\qquad \frac{8A_0^2s^2\kappa T}{|\gamma_2|^2}\ll1. \tag{28}$$

In works [3, 4], strictly speaking, such conditions are not sufficiently obeyed because of large values of the near dipole-dipole interaction parameter. In our case these conditions can be easily fulfilled by choosing the appropriate values of the parameters $\mathcal{W}_{01}$ and $\mathcal{W}_{02}$.

At first, let us consider the particular case when the $s=k=\rho=1$, $b_1=b_2=b$ and $\mathcal{W}_{01}=-\mathcal{W}_{02}=\mathcal{W}$. It is sufficient to prove that we are able to choose appropriate parameters Δ_1, Δ_2 and b to satisfy inequality $A_0^2\left(b,\Delta_1,\Delta_2,\mathcal{W}\right)=-\xi_1/\xi_3\ll1$. Solution of the equation $A_0^2\left(b,\Delta_1,\Delta_2,\mathcal{W}\right)=0$ with respect $\mathcal{W}$ is $\overline{\mathcal{W}}=2b/\left(\Delta_2-\Delta_1\right)$. Then the Taylor expansion of $A_0^2\left(b,\Delta_1,\Delta_2,\mathcal{W}+w\right)$ over small parameter $|w|\ll1$ gives us the following expression

$$A_0^2\left(b,\Delta_1,\Delta_2,\mathcal{W}+w\right)=\left(\frac{\Delta_1-\Delta_2}{2b}+\frac{\left(\Delta_1+\Delta_2\right)^2b}{2\left(\Delta_1-\Delta_2\right)}\right)w+O\left(w\right)^2. \tag{29}$$

It is clear that the conditions $A_0^2 > 0$ and $A_0^2 \ll 1$ can be easily satisfied by choosing the appropriate parameters Δ_1, Δ_2 and b. These conditions can be also met in the general case (without any assumptions introduced before).

Fig.2 demonstrates the evolution of the square amplitude of the pulse A_0^2 versus $\mathcal{W}_{01}$ and $\mathcal{W}_{02}$. One can see that there is a wide region of appropriate parameters where the inequalities (30) is valid (Fig.2 does not show the region of negative A_0^2).

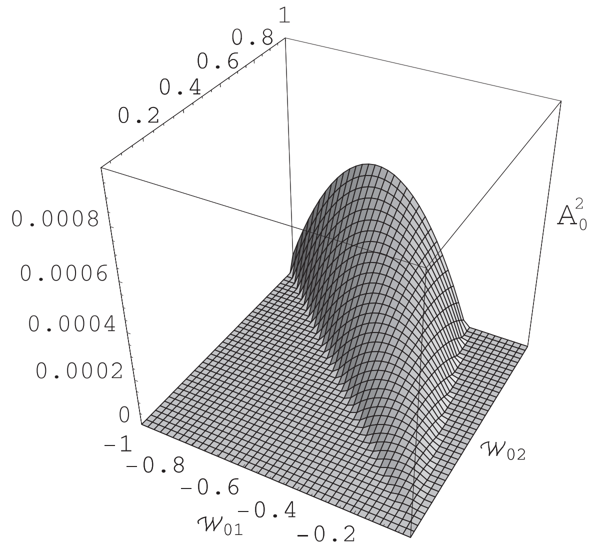

FIG. 2. The evolution of the square pulse amplitude A_0^2 versus stationary values of the population inversions $\mathcal{W}_{01}$ and $\mathcal{W}_{02}$ (when $\omega = 10^{14}s^{-1}$, $T_2^{(1)} = T_2^{(2)} = 10^{-10}s$, $l_{cav} = 4\cdot 10^{-4}cm^{-1}$, $N_1 = 10^{19}cm^{-3}$, $N_2 = 4.1\cdot 10^{19}cm^{-3}$, $\mu_1 = 2\cdot 10^{-19}$ CGSE, $\mu_2 = 10^{19}$ CGSE, $\Omega_1 = \Omega_2 = 10^{-7}s^{-1}$, $n_0 = 1.2$ and $\tau = 10^{-9}s$).

5. Stability analysis

One of the most important questions of studying the formation of solitons is the stability analysis. The small perturbations localized within the pulse envelop can destroy the stationary pulse. To investigate the stability of solitons with respect to small perturbations, one should linearize the equation(s) on the background of soliton solution and solve the spectral problem for the differential operator(s) [5, 6].

To find the stability condition, let us assume a small shift from the steady-state solution: $|E(z,u)| = A(u) + f(z,u)$. Then, after the linearization we can obtain the equation for the evolution of perturbation $f(z,u)$:

$$\frac{\partial f}{\partial z'} + \alpha\frac{\partial f}{\partial u} + f\left(\xi_1 + \xi_3\int\limits_{-\infty}^{u} A(u)^2\,du\right) + 2\xi_3 A(u)\int\limits_{-\infty}^{u} A(u) f\,du = 0. \tag{30}$$

We will seek for the solution of equation (30) in the factorized form

$$f\,(z,u) = f_0(u)\frac{\exp\left(\lambda u\right)}{\cosh\left(u\right)}\exp\left(2\lambda\xi_1 z'\right). \tag{31}$$

Substituting $f\,(z,u)$ into (30) and increasing the order of the differential equation, we can reduce the our problem to the eigenvalue one:

$$\frac{\partial^2 f_0(u)}{\partial u^2} - \left(\lambda^2 - \frac{2}{\cosh^2\left(u\right)}\right) f_0(u) = 0. \tag{32}$$

The solution of the eigenvalue problem gives the eigenfunction $f_0^{eigen} \propto 1/\cosh(u)$ with the eigenvalue $\lambda^2 = 1$, in other words $\lambda = \pm 1$. This means that the obtained solutions are unstable. The instability growth rate is proportional to the parameter ξ_1:

$$\overline{\lambda} = \lambda \frac{4\pi k_0 \mu_1^2 N_1 \tau}{n_0^2 \hbar} \xi_1 = \frac{4\pi k_0 \mu_1^2 N_1 T_1^{(2)}}{n_0^2 \hbar} \left(\frac{\sigma}{T} + \frac{\mathcal{W}_{01}}{|\gamma_1|^2} + s\kappa\rho \frac{\mathcal{W}_{02}}{|\gamma_2|^2} \right). \tag{33}$$

The smallness of the instability growth rate $\overline{\lambda}$ can be provided by the choice of appropriate parameters of active and passive media. Therefore the pulse can be regarded as an quasi-stable one over sufficiently short distances of propagation. Here we should also note that the smallness of the instability growth rate $\overline{\lambda}$ can be ensured if and only if there are absorbing and amplifying media.

6. Conclusion

Summing up, we can conclude that the incoherent solitons can obtained in dense resonant two-component media with some acceptable values of the constants of near dipole-dipole interaction of both media. In comparison with the single-component case, the two- or multi-component case allow one to overcome some arising difficulties.

The main one is the validation of cubic approximation (28), which guarantees the existence of the incoherent soliton solutions. As was shown in [3], the necessary requirement is a large value of near dipole-dipole interaction parameters in one-component dense resonant medium. The estimated large value of near dipole-dipole interaction parameter does not guarantee the validation of the cubic approximation. In case of two- or multi-component amplifying-absorber medium the condition (28) can be easily met by choosing the appropriate values of the media parameters.

The second difficulty is the minimization the value of the instability growth rate (33), which ensures the propagation of incoherent solitons over sufficiently short distances, keeping the practically stationary envelop. In is clear that we can easily minimize the value of the instability growth rate by varying the media parameters (see equation (33)). These considerations may be useful in choosing, for example, the length of a cavity of a soliton laser.

References

[1] McCall S.L., Hahn E.L., Phys. Rev. **183**(1969), 457.

[2] Bowden C.M., Dowling J.P., Phys. Rev. A **47** (1993), 1247.

[3] Afanas'ev A. A., Vlasov R. A., Cherstvyi A.G., ZETF **117**, (2000), 489.

[4] Afanas'ev A. A., Vlasov R. A., Khasanov O.K., Smirnova T.V., Fedorova O.M., J. Opt. Soc. Am. B **19**, (2002), 911.

[5] Kuznetsov E. A., Rubenchik A.M., Zakharov V.E., Phys. Rep. **142**, (1986), 105.

[6] Magyari E., Phys. Rev. A **31**, (1985), 1174.