Nonlinear Dynamics and Applications. Vol. 13 (2006) 127 - 132

Stability of Yang-Mills fields system in the homogeneous (anti-)self-dual vacuum field

V.I. Kuvshinov* and V.A. Piatrou†
Joint Institute for Power and Nuclear Research, Minsk, Belarus

Stability of Yang-Mills fields system in the vacuum field is investigated on the base of Toda criterion, construction of Poincare sections and calculation of the maximal Lyapunov exponents. The region of regular motion at low densities of energy is found in this model. The dependence of the critical density of energy of the order-chaos transition on value of the model parameter is obtained.

PACS numbers: 11.10.Lm
Keywords: chaos, Yang-Mills fields, homogeneous self-dual field, vacuum field

1. Introduction

Yang-Mills fields model is inherently nonlinear in contrast to electrodynamics. It is chaotic at any density of energy. This assumption was confirmed analytically and numerically [1–3]. Further research of spatially homogeneous field configurations [4] showed that Higgs field being included in the Yang-Mills system leads to order-chaos transition at some density of energy of classical gauge fields [5–7], while dynamics of gauge fields in the absence of Higgs field is chaotic at any density of energy. Classical Higgs fields regularize chaotic dynamics of classical gauge fields at densities of energy less than critical one and lead to appearance of order-chaos transition.

Chaos in Yang-Mills fields [8] and vacuum state instability in nonperturbative QCD models [9–11] are also considered in connection with confinement. Also it has been shown recently that interaction of the constant chromo-magnetic field with axial field could generate confinement [12]. These consequences indicate the existence of nonperturbative vacuum fields.

In this work the homogeneous (anti-)self-dual field [13] is regarded as a vacuum field in Yang-Mills fields model. Using some approximations we constructed two dimensional model in chromo-magnetic vacuum field and analyzed the stability of the model.

The system of Yang-Mills-Higgs fields in vacuum field under investigation was studied in previous paper [14]. It was demonstrated that there are stable and chaotic regions in parametric space. Their bounds were described analytically.

The system of Yang-Mills fields has an infinite number of degrees of freedom and it is too complicated to be investigated directly. In order to reduce the number of degrees of freedom, following other authors [15], we consider only spatially homogeneous fields. This model is a particular case of the general one. Spatially homogeneous field models allow one to investigate the main properties of inhomogeneous fields.

One more mechanism of the regularization of Yang-Mills fields is proposed in this work. Homogeneous (anti-)self-dual field being included in Yang-Mills model regularize Yang-Mills dynamics at low densities of energy. There is order-chaos transition at critical density of energy in the system.

*E-mail: V.Kuvshinov@sosny.bas-net.by
†E-mail: PiatrouVadzim@tut.by

2. Homogeneous (anti-)self-dual field

In this paper classical dynamics of SU(2) model gauge fields system is considered in the homogeneous (anti-)self-dual vacuum field. Various properties of this solution of the Yang-Mills equations in SU(2) theory were investigated originally by other authors [9, 16–18]. It was demonstrated that self-dual homogeneous field provides the Wilson confinement criterion [19]. Therefore this field is at least a possible source of confinement in QCD if it can be shown that such a field is a dominant configuration in the QCD functional integral.

Homogeneous self-dual field is defined by the following expressions [13]:

$$B_\mu^a = Bn^a b_{\mu\nu} x_\nu,$$

$$F_{\mu\nu}^a = -2Bn^a b_{\mu\nu},$$

where B - value of the field strength, n^a characterizes the direction of the field in color space, tensor $b_{\mu\nu}$ - the direction in coordinate space.The latter has the following properties

$$b_{\mu\nu} = -b_{\nu\mu}, \qquad b_{\mu\nu} b_{\mu\rho} = \delta_{\nu\rho},$$

$$\widetilde{b}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} b_{\alpha\beta} = \pm b_{\mu\nu},$$

where positive and negative signs in last expression correspond, respectively, to self-dual and anti-self-dual cases.

The color vector n^a points in some fixed direction which can be chosen as $(n^1, n^2, n^3) = (0, 0, 1)$ [16, 18]. The direction of the nonperturbative field in coordinate space can be chosen arbitrarily. We will assume that it is directed along Z axis [16, 18, 19]. The tensor $F_{\mu\nu}$ will be the following:

$$F_{\mu\nu} = \begin{pmatrix} 0 & -B & 0 & 0 \\ B & 0 & 0 & 0 \\ 0 & 0 & 0 & \pm B \\ 0 & 0 & \mp B & 0 \end{pmatrix}$$

It can be also rewritten in the following form

$$\mathbf{B} = (B_1, B_2, B_3) = (0, 0, B).$$

3. Model potential of the system

The Lagrangian of SU(2) gauge theory in Euclidean metrics is

$$L = -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a,$$

where $G_{\mu\nu}^a$ is a field tensor which has the following form:

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc} A_\mu^b A_\nu^c.$$

In last expression $A_\mu^a, a = 1, 2, 3$ are the three non-abelian Yang-Mills fields and g denotes the coupling constant of these fields.

We consider the fluctuations around vacuum homogeneous self-dual field. Self-dual field is regarded as external and it is taken into account by substituting modified vector potential in the Yang-Mills Lagrangian

$$A_\mu^a \to A_\mu^a + B_\mu^a$$

where A_μ^a is the fluctuation to the vacuum field B_μ^a .

We use the gauge:

$$A_4^a = 0,$$

and consider spatially homogeneous field configurations [15]

$$\partial_i A_\mu^a = 0, \qquad i = 1..3.$$

Our model of Yang-Mills fields in (anti-)self-dual field is constructed in Euclidean space. In order to analyze the model by using analytical and numerical methods we should switch to Minkowski space. We consider chromo-magnetic model. Thus we put chromo-electric field is equal to zero. If $A_1^1 = q_1$, $A_2^2 = q_2$ and the other components of the perturbative Yang-Mills fields are equal to zero, the potential of the model is:

$$V = \frac{1}{2}g^2 q_1^2 q_2^2 + \frac{1}{2}H^2 - gHq_1q_2 + \frac{1}{8}g^2H^2(x^2q_1^2 + y^2q_2^2), \tag{1}$$

where H - nonperturbative chromo-magnetic field strength, x and y - coordinates which play the role of the parameters, q_1 and q_2 - field variables.

4. Stability of the model

4.1. Toda criterion

At first stability of the model is investigated using well known technique based on the Toda criterion of local instability [20, 21]. This criterion being applied for potential (1) could give us the value of the critical density of energy of order-chaos transition in the system. The dependence of this value on the model parameter $s = gHxy$ is shown on the FIG.1. The value of the minimal energy in the system as a function of the model parameter is also demonstrated on this figure (thick line).

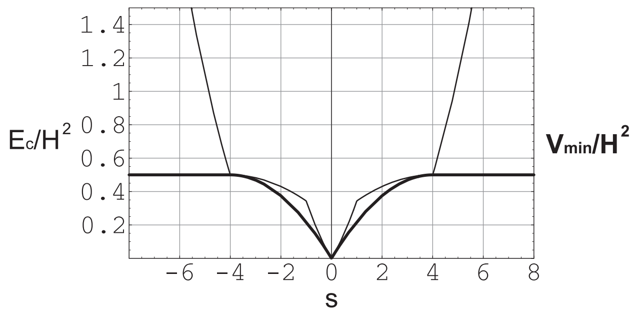

FIG. 1. Critical density of energy of order-chaos transition (thin line) and minimal energy in the system (thick line) as a function of the model parameter $s = gHxy$.

From the comparison of the minimal and critical energy one could obtain the following consequences:

a) These energies are close to each other for $s \in (-4, 4)$ and the system is chaotic at any density of energy.

b) The critical density of energy is rather greater than minimal one and the system is regular at small densities for $s \in (-\infty, -4)$ or $s \in (4, \infty)$.

These consequences will be checked using numerical methods in next subsection.

4.2. Numerical calculations

The system is investigated using Poincare sections and Lyapunov exponents for wide range of model parameter values. These numerical methods could indicate global regular regimes of motion whereas Toda criterion reveals only the local chaotic properties of the trajectories [22]. Thus numerical methods are more precise for stability analysis. Results of the numerical

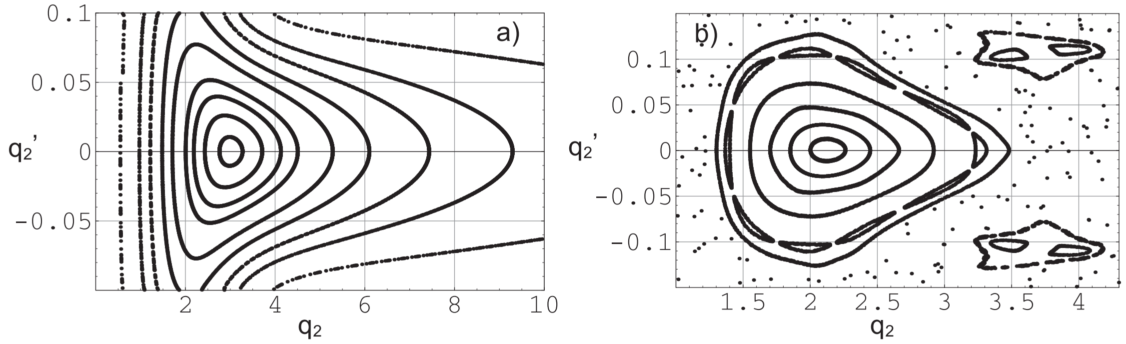

FIG. 2. Poincare sections for two dimensional Yang-Mills system in the vacuum field for $s = 0, H = 1, (a)E = 0.005$ and $(b)E = 0.15$.

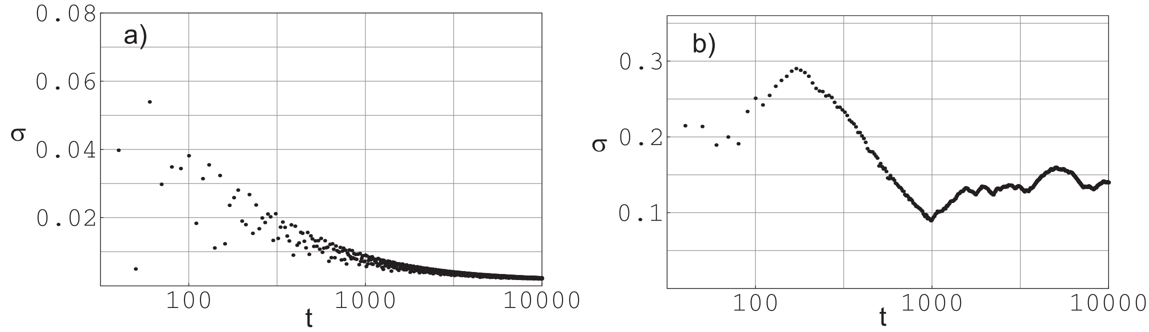

FIG. 3. Maximal Lyapunov exponents for two dimensional Yang-Mills system in the vacuum field for $s = 0, H = 1, (a)E = 0.15$ and $(b)E = 0.68$.

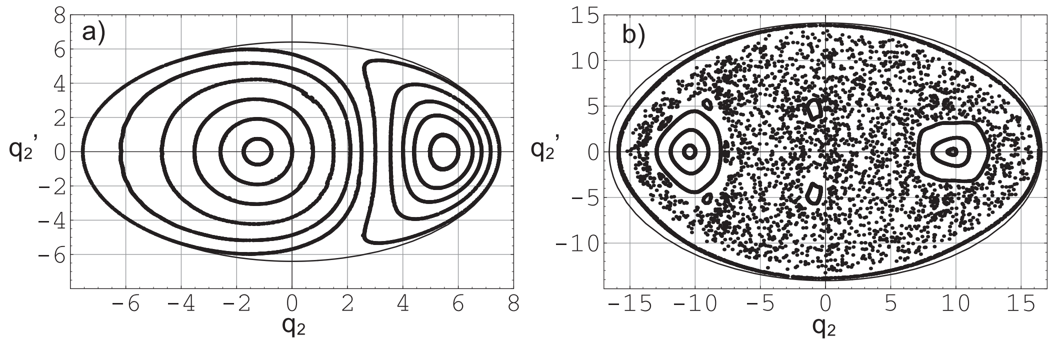

FIG. 4. Poincare sections for two dimensional Yang-Mills system in the vacuum field for $s = 25.5, g = 0.1, H = 1, x = 15, y = 17, (a)E = 21$ and $(b)E = 100$. Thin line - border of the phase space.

calculations for the system with model parameter $s = 0$ are shown on the FIG.2 and FIG.3. It is seen that system is regular at small densities of energy contrary to Toda criterion. The critical energy is equal to the energy of the vacuum field $E_c = E_{vac} = \frac{1}{2}H^2$. The system is regular at energies $E < E_{vac}$ and chaotic otherwise.

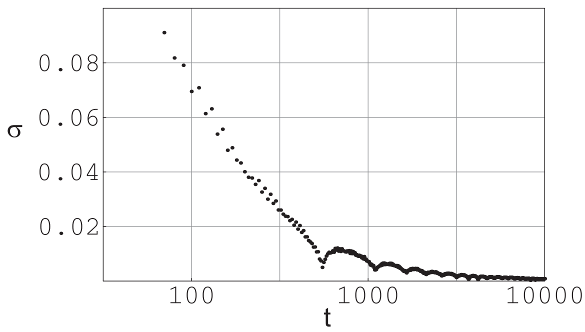

FIG. 5. Maximal Lyapunov exponents for two dimensional Yang-Mills system in the vacuum field for $g = 0.1, H = 1, x = 15, y = 17$ and $E = 100$.

In the range of the large model parameter values numerical methods give us the following results (FIG.4 - FIG.5). It follows that system is regular for high values of energy ($E_c >> E_{vac}$) as it was shown by Toda criterion. But the system is chaotic at value of energy greater than critical one.

It is seen that Toda criterion rather good describes the region of the large values of model parameter s and fails for $s \in (-4, 4)$.

Numerical calculations have shown that there is a region of regular motion at low densities of energy in our system at any value of the model parameter. Therefore, homogeneous (anti-)self-dual field regularizes chaotic dynamics of Yang-Mills fields system.

5. Conclusions

In this work - homogeneous (anti-)self-dual field - was identified to regularize the chaotic Yang-Mills dynamics. Yang-Mills model with this type of the vacuum field has the region of regular motion at low densities of energy. There is order-chaos transition in the system for all range of model parameters. The critical density of energy of this transition is equal to the vacuum field energy for small parameters and significantly greater that one for large parameter values. These consequences were obtain on the base of analytic and numerical calculations.

Acknowledgements

This work is supported by Belarusian Republican Foundation for Fundamental Research.

References

[1] S. G. Matinyan, G. K. Savvidy and N. G. Ter-Arutunyan-Savvidy. Classical Yang-Mills mechanics. Nonlinear color oscilations. Sov.Phys.JETP **53**, 421 (1981).

[2] B. V. Chirikov and D. L. Shepelyansky. Stochastic oscillations of classical Yang-Mills fields. JETP Lett. **34**, no. 4, 163–166 (1981).

[3] G. K. Savvidy. The Yang-Mills classical mechanics as a Kolmogorov K-system. Phys. Lett. **B130**, no. 5, 303–307 (1983).

[4] G. Z. Baseyan, S. G. Matinyan and G. K. Savvidy. Nonlinear plane waves in massless Yang-Mills theory. Pis'ma Zh. Eksp. Teor. Fiz. **29**, no. 10, 641–644 (1979).

[5] S. G. Matinian, G. K. Savvidy and N. G. Ter-Arutunian. Stochastisity of Yang-Mills mechanics and its elimination by Higgs mechanism. Pisma Zh.Eksp.Teor.Phys. **34**, no. 11, 613–616 (1981).

[6] G. Savvidy. Classical and quantum mechanics of nonabelian gauge fields. Nucl.Phys. **B246**, 302–359 (1984).

[7] G. P. Berman, Y. I. Mankov and A. F. Sadreyev. Stochastic instability of classical homogeneous SU(2)$\bigotimes$U(1) fields with spontaneously broken symmetry. Zh. Eksp. Teor. Fiz. (JETP) **88**, no. 3, 705–714 (1985).

[8] V. I. Kuvshinov and A. V. Kuzmin. Towards chaos criterion in quantum field theory. Phys. Lett. A. **296**, 82–86 (2002).

[9] G. K. Savvidy. Infrared instability of the vacuum state of gauge theories and asymptotic freedom. Phys.Lett. **71B**, 133–134 (1977).

[10] N. K. Nielsen and P. Olesen. An unstable Yang-Mills field mode. Nucl. Phys. **B144**, 376–396 (1978).

[11] J. Ambjørn, N. K. Nielsen and P. Olesen. A hidden Higgs lagrangian in QCD. Nucl. Phys. **B152**, 75–96 (1979).

[12] P. Gaete and E. Spallucci. Confinement effects from interacting chromo-magnetic and axion fields. J. Phys. A **39**, 6021–6030 (2006).

[13] I. A. Batalin, S. G. Matinian and G. K. Savvidy. Vacuum polarization by a source-free gauge field. Sov. J. Nucl. Phys. **26**, 214 (1977).

[14] V. I. Kuvshinov and V. A. Piatrou. Stability of Yang-Mills-Higgs field system in the homogeneous self-dual vacuum field. Nonlin.Phenom.Complex Syst. **8**, no. 2, 200–205 (2005).

[15] B. V. Chirikov and D. L. Shepelyansky. Dynamics of some homogeneous models of classical Yang-Mills fields. Sov. J.Nucl. Fiz. **36**, no. 6, 908–915 (1982).

[16] H. Leutwyler. Vacuum fluctuations surrounding soft gluon fields. Phys.Lett. **96B**, 154–158 (1980).

[17] P. Minkowski. On the ground-state expectation value of the field strength bilinear in gauge theories and constant classical fields. Nucl.Phys. **B177**, 203–217 (1981).

[18] H. Leutwyler. Constant gauge fields and their quantum fluctuations. Nucl.Phys. **B179**, 129–170, (1981).

[19] G. V. Efimov, A. C. Kalloniatis and S. N. Nedelko. Confining properties of the homogeneous self-dual field and the effective potential in SU(2) Yang-Mills theory. Phys.Rev. **D59**, 014026 (1999).

[20] M. Toda. Instability of trajectories of the lattice with cubic nonlinearity. Phys.Lett. **A 48**, no. 5, 335–336 (1974).

[21] L. Salasnich. Quantum signature of the chaos-order transition in a homogeneous SU(2) Yang-Mills-Higgs system. Phys.Atom.Nucl. **61**, 1878–1881 (1998).

[22] G. Benettin, R. Brambilla and L. Galgani. A comment on the reliability of the toda criterion for the existence of a stochastic transition. Physica **A87**, no. 2, 381–390 (1977).