Nonlinear Dynamics and Applications. Vol. 13 (2006) 121 - 126

Chaos assisted instanton tunneling in monochromatically driven double-well potential

V.I. Kuvshinov* and A.V. Kuzmin†
Joint Institute of Power and Nuclear Research - Sosny,
National Academy of Sciences of Belarus
Acad. Krasina Str. 99, 220109 Minsk, Belarus

A.V. Lougovski
Belarussian State University
Independence ave. 4,
220000 Minsk, Belarus

Chaos assisted instanton tunneling is considered on the example of the double-well potential driven by a monochromatic force. The expression for the ground quasienergy doublet splitting determining the tunneling oscillations frequency is derived in the framework of the instanton technique exploiting chaotic instanton solutions.

PACS numbers: 03.65.Xp, 05.45.Mt
Keywords: instanton, chaos assisted tunneling

1. Introduction

Tunneling as inherently quantum phenomenon attracts much attention [1]. Its connection with classical chaos in semiclassical regime has been discussed [2, 3]. A number of works were devoted to semiclassical chaos assisted tunneling between symmetry related KAM-tori in systems having mixed dynamics (well developed chaotic region coexists in phase space with regular islands) [4–6]. To describe chaos assisted tunneling in systems demonstrating mixed dynamics the multi-level model Hamiltonian, primarily proposed in [2], is often used [7]. Less attention has been payed to semiclassical tunneling in KAM systems (chaotic region is not widespread) [8]. Instanton technique [9, 10] was used in a very few works [11].

In this work we consider one-dimensional quantum system with double-well potential affected by small periodic in time perturbation. We use methods developed to describe chaos in classical Hamiltonian systems to investigate essentially quantum phenomenon of tunneling. It is achieved in the framework of instanton technique, where solutions of Euclidian equations of motion (instantons) play dominating role. For the systems driven by the periodic in time perturbation the energy is no more the exact integral of motion and the language of quasienergies has to be used instead [13, 14]. Energy approximately can be considered as an adiabatic invariant [15]. We study properties of the chaotic instanton solutions and calculate the ground quasienergy doublet splitting.

There are papers devoted chaos assisted tunneling where some analytical predictions for billiard systems basing on the path integral formulation of quantum mechanics are made [3]. Distinguishing feature of our work is analytical predictions for the system with smooth potential and adoption for this purpose instanton technique [9].

*E-mail: v.kuvshinov@sosny.bas-net.by
†E-mail: avkuzmin@sosny.bas-net.by

Hamiltonian of the considered model is given by the expression:

$$H(x,p) = \frac{p^2}{2m} + \frac{m\,\omega^2}{8\,a^2}(x^2-a^2)^2 - \epsilon\,x\,\cos\nu t. \quad (1)$$

Corresponding Schroödinger equation can be solved in the framework of the Floquet formalism. However, in this work we exploit the chaotic instanton technique developed in the previous papers [16–18].

2. Chaotic instanton solutions

Hamiltonian of the system (1) in the imaginary time has the form:

$$\begin{aligned} H(x,p) &= H_0 + V \\ H_0 &= \frac{p^2}{2m} - \frac{m\omega^2}{8a^2}(x^2-a^2)^2 \\ V &= \epsilon x \cos\nu\tau, \end{aligned} \quad (2)$$

where H_0 denotes non-perturbed Hamiltonian of the system in Euclidean time and V is Euclidean perturbation. In order to apply the chaotic instantons technique one has to calculate the width of the stochastic layer replacing the separatrix of the non-perturbed system when the potential V is applied [16–18].

Primarily let us consider Hamiltonian H_0 without driving force. In this case the separatrix do exists in the classical phase space of the system. As the result of the monochromatic driving force application the stochastic layer replacing the separatrix appears. We restrict ourselves by consideration of parameters values corresponding to the case of the homogeneous stochastic layer.

For the non-perturbed system the instanton solution and its time derivative are given by the expressions:

$$x(t) = -a\,\tanh[\frac{\omega(\tau-\tau_0)}{2}] \quad (3)$$

$$\dot{x}(t) = -\frac{\omega a}{2}\,\frac{1}{\cosh^2[\frac{\omega(\tau-\tau_0)}{2}]}. \quad (4)$$

Corresponding action equals

$$I = I(H) = \frac{2}{\pi}\int_0^{x_0}(2m\,(H + \frac{m\,\omega^2}{8\,a^2}(x^2-a^2)^2))^{1/2}dx, \quad (5)$$

where the turning point x_0 is defined by the condition

$$H + \frac{m\,\omega^2}{8\,a^2}(x_0^2-a^2)^2 = 0. \quad (6)$$

The frequency of the nonlinear oscillation is

$$\omega(H) = \frac{d\,H(I)}{d\,I} = (\frac{d\,I(H)}{d\,H})^{-1} \quad (7)$$

$$|\omega'| = |\frac{d\,\omega}{d\,I}| = |\frac{d\,\omega}{d\,H}\frac{d\,H}{d\,I}| = \omega|\frac{d\,\omega}{d\,H}| \quad (8)$$

Thus

$$|\frac{\omega'}{\omega^3}| \sim \frac{a\sqrt{m}}{H\pi}. \tag{9}$$

As the result of the direct but lengthy calculations the following expression for the local instability parameter is obtained

$$K = \frac{\pi\,\epsilon\,\nu\,|\omega'|}{\omega^3} C_0, \tag{10}$$

where

$$C_0 = |\int \frac{\partial\,V}{\partial\,x} \dot{x} \sin\nu\tau| \tag{11}$$

and $V(x) = x$ in the case considered. The substitution of the (3) into the (11) yields

$$C_0 = |Im(\int\limits_{-\infty}^{\infty} -\frac{\omega a}{2} \frac{1}{\cosh^2[\frac{\omega(\tau-\tau_0)}{2}]} \exp[\nu\tau])|. \tag{12}$$

Calculating this integral we obtain

$$C_0 = \omega a \pi \nu \exp[-\frac{\pi\nu}{\omega}]. \tag{13}$$

Condition $K \geq 1$ corresponds to the chaotic behavior of the system and lets us to estimate the width of the stochastic layer

$$\Delta H = \frac{\epsilon\omega\sqrt{m}\pi a^2 \nu^2 \exp[-\frac{\pi\nu}{\omega}]}{2}, \tag{14}$$

which is proportional to the perturbation strength.

3. Ground quasienergy doublet splitting

The tunneling amplitude can be expressed as the following path integral in the imaginary time:

$$A = \langle x_f | e^{-H\tau_0} | x_i \rangle = N \int [Dx] e^{-S}, \tag{15}$$

where H is Hamiltonian and $e^{-H\tau_0}$ is the evolution operator of the system in Euclidean time, S - the action, N is a normalization factor, $[Dx]$ means integration over all functions $x(t)$ with boundary conditions $x(-\tau_0/2) = x_i$, $x(\tau_0/2) = x_f$.

Now we obtain Euclidean action. We have to consider both the separatrix solutions (instanton and anti-instanton) and the solutions bellow the separatrix. Thus we obtain

$$S = \int\limits_{x^-}^{x^+} (2m(-H + \frac{m\,\omega^2}{8\,a^2}(x^2 - a^2)^2))^{1/2} dx, \tag{16}$$

where

$$x^{\pm} = \pm a\sqrt{1 - \sqrt{\frac{8H}{m\omega^2 a^2}}}. \tag{17}$$

In order to simplify subsequent calculations we perform Taylor expansion of the result of integration and keep the first non-vanishing term:

$$S = \frac{4}{3\omega}(\frac{m\omega^2 a^2}{2} - H). \tag{18}$$

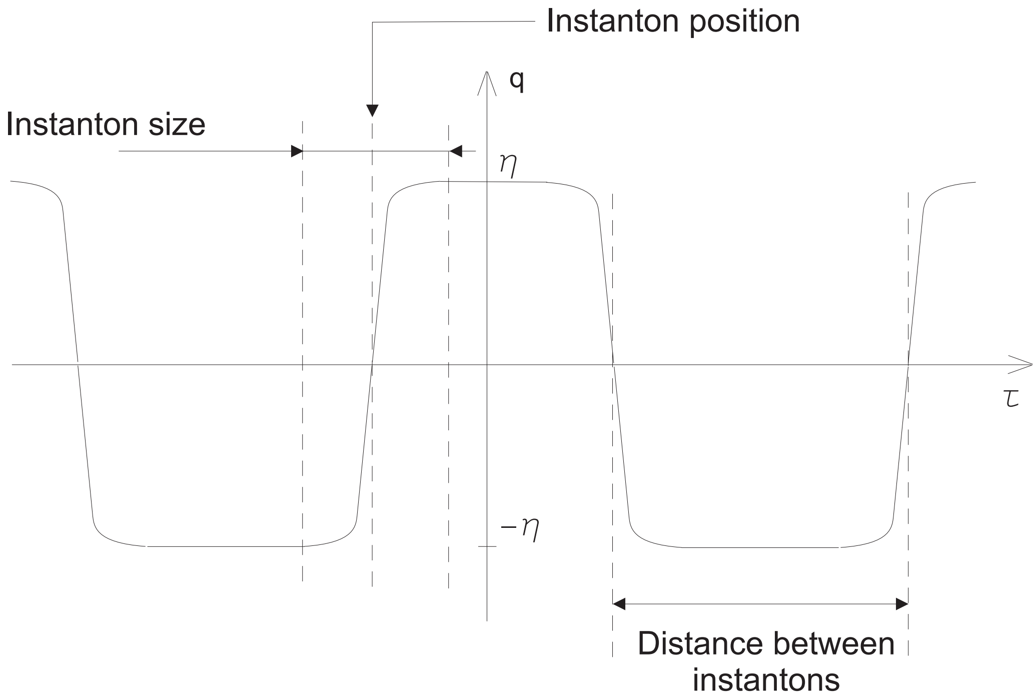

FIG. 1: Example of the multi-instanton configuration

Exploiting the standard instanton technique [19] for the calculation of the path integral (15) one obtains:

$$[Dx] = \prod_n \frac{dc_n}{\sqrt{2\pi}},$$
$$\langle x_f | e^{-H\tau_0} | x_i \rangle = N e^{-S} \prod_n {\varepsilon_n}^{-1/2}, \tag{19}$$

where ε_n are eigenvalues of the operator $-\frac{d^2}{d\tau^2} + V''(X)$. Then we change the integration over c_0 to τ_o. That leads to the additional coefficient $\sqrt{S_0}$. Then we find final expression for the one instanton contribution:

$$\langle x_f | e^{-H\tau_0} | x_i \rangle = (\sqrt{\frac{m\,\omega}{\pi}}\, e^{-m\,\omega\tau_0/2})(\sqrt{\frac{6}{\pi}}\,\sqrt{S}\,e^{-S}) m\omega d\tau_0. \tag{20}$$

After that we have to take into account the contributions of the multi-instanton configurations (see fig. 1). It should be mentioned that the sequence order of instantons and anti-instantons in this problem is not arbitrary. They are considered as alternate. So the configuration is the following: instanton then follows anti-instanton then again instanton and so on. As a result we obtain [19]:

$$\sqrt{\frac{m\omega}{\pi}}\, e^{-m\omega\tau_0/2}\, r^n \int_{-\tau_0/2}^{\tau_0/2} m\omega d\tau_n \int_{-\tau_0/2}^{\tau_n} m\omega d\tau_{n-1} \ldots \int_{-\tau_0/2}^{\tau_2} m\omega d\tau_1 =$$
$$= \sqrt{\frac{m\omega}{\pi}}\, e^{-m\omega\tau_0/2}\, r^n \frac{(m\omega\tau_0)^n}{n!}, \tag{21}$$

where r is the instanton density. The value of r is given by the following equation

$$r = \sqrt{\frac{6}{\pi}}\,\sqrt{S}\,e^{-S}. \tag{22}$$

Since we consider the tunneling between different wells, it is necessary to sum expression (21) only over odd n. Finally we have

$$\langle -a|e^{-H\tau_0}|a\rangle = \sum_{n=1,3,\ldots} \sqrt{\frac{m\omega}{\pi}}\, e^{-m\omega\tau_0/2}\, \frac{(m\omega\tau_0 r)^n}{n!}. \tag{23}$$

In order to derive the tunneling amplitude and ground levels energies one should substitute the action (18) to the amplitude (23) and integrate it over the energy H in limits of stochastic layer width. Thus we obtain

$$A = \sum_{n=1,3,\ldots} \int\limits_0^{\Delta H} \frac{(m\omega\tau_0\sqrt{\frac{6}{\pi}}\,\sqrt{\frac{4}{3\omega}(\frac{m\omega^2a^2}{2}-H)}\; e^{-\frac{4}{3\omega}(\frac{m\omega^2a^2}{2}-H)})^n}{n!} \times$$
$$\times\sqrt{\frac{m\omega}{\pi}}\, e^{-m\omega\tau_0/2}\, dH. \tag{24}$$

Where ΔH is given by (14). Calculation of this integral and subsequent simplifications give us the following expressions for lowest energy levels in the wells:

$$E_0 = \frac{m\omega}{2}(1 - 4a\exp\frac{2(a^2\, e^{-\frac{\pi\nu}{\omega}}\sqrt{m}\pi\epsilon\nu^2 - a^2m\omega)}{3} \times$$
$$\times\sqrt{\frac{m\omega}{\pi} - \, e^{-\frac{\pi\nu}{\omega}}\sqrt{m}\epsilon\nu^2} \tag{25}$$

$$E_1 = \frac{m\omega}{2}(1 + 4a\exp\frac{2(a^2\, e^{-\frac{\pi\nu}{\omega}}\sqrt{m}\pi\epsilon\nu^2 - a^2m\omega)}{3} \times$$
$$\times\sqrt{\frac{m\omega}{\pi} - \, e^{-\frac{\pi\nu}{\omega}}\sqrt{m}\epsilon\nu^2} \tag{26}$$

Thus the ground quasienergy doublet splitting is given by the formula:

$$\Delta E = 4m\omega a\exp\left(\frac{2(a^2\, e^{-\frac{\pi\nu}{\omega}}\sqrt{m}\pi\epsilon\nu^2 - a^2m\omega)}{3}\right)\sqrt{\frac{m\omega}{\pi} - \, e^{-\frac{\pi\nu}{\omega}}\sqrt{m}\epsilon\nu^2}. \tag{27}$$

This splitting (divided by Plank constant) equals the frequency of the tunneling oscillations between symmetric and anti-symmetric superpositions of the two ground Floquet states localized in the distinct potential wells. It is seen that the frequency exponentially grows when the perturbation strength ϵ increases. This conclusion coincide with the result obtained for the system having periodic in space potential [16, 17].

4. Conclusions

In this work the instanton technique exploiting chaotic instanton solutions was applied to obtain the ground quasienergy doublet splitting and the frequency of the corresponding tunneling oscillations between minima of the double-well potential driven by the external monochromatic force. Exponential dependence of the frequency on the driving strength was derived. It coincides with the conclusion made for the system with periodic in space potential [16–18]. It demonstrates the fact that the exponential repulsion of the low quasienergy levels is the universal phenomenon and does not depend on the potential of the system under consideration or on the form of the periodic perturbation, but it is exclusively determined by the characteristics of the chaotic instanton solutions, namely the corresponding stochastic layer width. It should be emphasized that the conclusion made valid in the case when the stochastic layer is narrow and does not have the significant islands of stability inside itself. As well the absence of the low quasienergy level crossings is assumed.

Acknowledgements

This work is partially supported by Belarusian Republican Fund for Fundamental Research.

References

[1] A.E. Faraggi and M. Matone, Phys.Lett. B**445** (1999) 357; F. Grossmann, Phys.Rev.Lett. **85** (2000) 903.
[2] O. Bohigas, S. Tomsovic, and D. Ullmo, Phys.Rep. **223** (1993) 43.
[3] S.D. Frischat and E. Doron, Phys.Rev. E **57** (1998) 1421; E. Doron and S.D. Frischat, Phys.Rev.Lett. **75** (1995) 3661.
[4] M. Latka, P. Grigolini, and B.J. West, Phys.Rev. A **50** (1994) 1071.
[5] B. Mirbach and H.J. Korsch, Phys.Rev.Lett. **75** (1995) 362.
[6] A. Mouchet, C. Miniatura, R. Kaiser, B. Gremaud, and D. Delande, E-print: nlin.CD/0012013 (2000).
[7] S. Kohler, R. Utermann, and P. Hanggi, E-print: quant-ph/9804071 (1998).
[8] M.J. Davis, J.Phys.Chem. **92** (1988) 3124.
[9] A. Belavin, A. Polyakov, A. Schwarz, and Yu. Tyupkin, Phys.Lett. B**59**, 85 (1975); G. 't Hooft, Phys.Rev.Lett. **37**, 8 (1976), Phys.Rev. D **14**, 3432 (1976).
[10] J. Zinn-Justin, U.D. Jentschura, Ann.Phys. **313** (2004) 197; Ann.Phys. **313** (2004) 269.
[11] K.-I. Aoki, A. Horikoshi, M. Taniguchi, and H. Terao, E-print: hep-th/9812050 (1998); H. Jirari, H. Kroger, X.Q. Luo, K.J.M. Moriarty, and S.G. Rubin, E-print: quant-ph/9910116 (2001).
[12] A.J. Lichtenberg, M.A. Lieberman, *Regular and Stochastic Motion,* Springer Publishing, 1983.
[13] J.H. Shirley, Phys. Rev. vol.138 (1965) B979.
[14] S. Chu, Adv. Chem. Phys. **73** (1989) 739.
[15] G.M. Zaslavsky and R.Z. Sagdeev, *Introduction in nonlinear physics*, Moskow, 1988, chapter 5 (in Russian).
[16] V.I. Kuvshinov, A.V. Kuzmin, Physics of Elementary Particles and Atomic Nuclei, vol.36 (2005) 183.
[17] V.I. Kuvshinov, A.V. Kuzmin, R.G. Shulyakovsky, Phys.Rev. E, vol.67 (2003) 015201(R).
[18] V.I. Kuvshinov, A.V. Kuzmin, R.G. Shulyakovsky, Acta Phys. Pol. B, vol.33 (2002) 1721.
[19] A.P. Vainshtein, V.I. Zakharov, V.A. Novikov, M.A. Shifman, Usp. Fiz. Nauk vol.136 (1982) 553 (in Russian).