Formation of azimuthally and radially polarized Bessel light beams using one-dimensional photonic nonlinear crystals

S.N. Kurillkina^{*} and V.N. Belyi and N.S. Kazak

B.I.Stepanov Institute of Physics of the National Academy of Science of Belarus 68 Nezavisimosti Ave., 220072, Minsk, Belarus

A method is suggested for forming and switching azimuthally φ and radially ρ polarized Bessel light beams (BLB) using a one-dimensional photonic crystal with a nonlinear layer in the unit cell. The method is based on splitting the spectral dependence of transmission (reflection) of a vectoral BLB, propagating into periodic medium, on two curves corresponding to ρ and φ polarized light beams. Owing to this, one can realize the transmission of BLB with the determined type of polarization and reflection of another polarized beam. The conditions are established of the polarization change for transmitted (reflected) BLB, using controlling pulse. The elaborated method permits one not only to separate beams with the azimuthal and radial polarization, but to form their any superposition. This is achieved by the change of the intensity of the controlling pulse which results in rebuilding the transmission spectra of the and beams and, as a consequence, in the control over the ellipticity of transmitted and reflected Bessel beams. High-effective and compact optical switches of polarized Bessel light beams are proposed.

PACS numbers: 78.67.Pt+42.65.Ky

Keywords: one-dimensional photonic crystal, photonic band gap, Bessel light beam

1. Introduction

One of the important directions in optics of Bessel light beams (BLBs) is elaborating the methods for formation of the beams with established polarization: azimuthal or radial one. Focusing radially (ρ) and azimuthally (φ) polarized BLBs permits one to achieve higher axial concentration of electric and magnetic fields as compared with the case of linear or circular polarization [1]. Hence, these beams are perspective in photolithography, confocal microscopy, in recorders and sensors. Radially and azimuthally polarized Bessel light beams with a large angle of conicity and their superposition have a great value of the transverse gradient of intensity and can reconstruct the space configuration of the field. So, they are promising for keeping microparticles and molecules and controlling their movement [2-5]. Owing to total cylindrical symmetry φ and ρ polarized BLBs are optimal for application in different probing schemes for cylindrical objects [6,7]. Last time some methods of obtaining radially and azimuthally polarized light fields are proposed. The simplest method consists in the use of polarizing property of conical surface at the incidence of the light under Brewster' angle [8]. But for complete separation of φ and ρ polarized components in this case it is necessary repeated passing

^{*}E-mail: s.kurilkina@ifanbel.bas-net.by

the light trough conical surface. It complicates the scheme. ρ polarized field can be obtained by the method of mode converter owing to forming superposition of two orthogonally polarized Hermit- Gaussian modes of the first order [9]. It has been proposed also a series of intracavity methods for formation of φ and ρ polarized modes (see, for example, [10-13]). Recently it has been elaborated a new method for formation of azimuthally and radially polarized BLBs based on separation of circularly polarized Bessel beam with the help of layered- periodical medium having imperfect crystalline layer [14]. Nevertheless, creation of simple, compact and inexpensive polarizer for formation of φ and ρ polarized Bessel beams remains important task. In the report we proposed one of the methods of its solution based on the use of one-dimensional photonic crystals having nonlinear layer into a unit cell.

2. Description of Bessel light beam transformation into one-dimensional photonic crystal

Let us consider a one-dimensional photonic crystal. Its special case is a multi-layer stack containing N periods of dielectric materials. Each period has layers with a high n_1 and a low n_2 index of refraction. Such a structure exhibits an electro-magnetic stop band for photon propagation over a wide band of frequencies. We will consider now a Bessel beam that impinges on such a structure with its frequency near the gap edge.

Let us choose a coordinate system for which Z axis is collinear to direction of periodicity. First of all we will consider the peculiarities of transformation of given Bessel beam into a unit cell with a thickness d. For given periodical structure fields of φ and ρ polarization are own waves and, consequently, it is possible their separate consideration. So, we will present electric vector (for example for φ BLB) in the medium as a superposition of vectors corresponding to right-going and backward propagating beams, labeled plus and minus, respectively [15]:

$$\overrightarrow{E} = \begin{pmatrix} E^+ \\ E^- \end{pmatrix}.$$
 (1)

Here $E^{\pm} = f^{\pm}(z) \exp(\pm ikz)$, and $\overrightarrow{E}(0) = M \overrightarrow{E}(d)$, *M* is transfer matrix. Taking into account the boundary conditions

$$\vec{E}(0) = \begin{pmatrix} 1\\r \end{pmatrix}, \vec{E}(d) = \begin{pmatrix} t\\0 \end{pmatrix},$$
(2)

where r, t are respectively complex coefficients of reflection and transmission of the unit cell, it is possible to obtain the expression for a transfer matrix [15]:

$$M = \begin{pmatrix} 1/t & r^*/t^* \\ r/t & 1/t^* \end{pmatrix} .$$
(3)

In the case of periodical medium with the unit cell forming two dielectrics the transfer matrix will be the product of matrices of only two types.

$$M = \Delta_{12} \Pi(-p_2) \Delta_{21} \Pi(-p_1).$$
(4)

The first is a matrix Δ that transfers the field amplitude left to right across the $n_i \rightarrow n_j$ interface.

$$\Delta_{ij} = \begin{pmatrix} a_{ij}^+ & a_{ij}^- \\ a_{ij}^- & a_{ij}^+ \end{pmatrix},\tag{5}$$

 $a_{ij}^+ = 1/t_{ij}, a_{ij}^- = r_{ij}/t_{ij}, t_{ij}, r_{ij}$ are the $n_i \rightarrow n_j$ interface transmission and reflection coefficients.

The second matrix included in (4) is a propagation one:

$$\Pi(p_i) = \begin{pmatrix} e^{ip_i} & 0\\ 0 & e^{-ip_i} \end{pmatrix},\tag{6}$$

where $p_i = \frac{\omega}{c} n_i \cos \gamma_i$, γ_i is the half- angle of conicity of BLB propagating in the medium with n_i refractive index. Using the boundary conditions, one may obtained the following expressions for the case of φ BLB transformation on the border of two media:

$$t_{ij} = \frac{2n_i \cos \gamma_i}{n_i \cos \gamma_i + n_j \cos \gamma_j}, r_{ij} = \frac{n_i \cos \gamma_i - n_j \cos \gamma_j}{n_i \cos \gamma_i + n_j \cos \gamma_j}.$$
(7)

Corresponding expressions for ρ BLB one may obtain from (7) by replacement $n_i \to 1/n_i$. Let us consider now the peculiarities of BLB scattering in unlimited periodical medium when n(Z) = n(Z + d). It is known that peculiarities of light waves transformation in given medium are described by Bloch functions changing only phase on the value $\mu_b = exp(\pm i\beta)$ from cell to cell. Here is determined as eigenvalue of transfer matrix and, how it follows from (3):

$$\cos\beta = Re\left(1/t\right),\tag{8}$$

Expression (8) is dispersion equation for unlimited periodical medium. In the case of the medium, containing N unit cells and limited by material with refraction index n_0 , the transfer matrix M', connecting light field at the entrance and exit of the structure, can be presented in the form $M' = \Delta_{01} M^N \Delta_{10}$, where matrix Δ_{ij} is determined by expressions (5), (7). If we write transfer matrix M' similarly to (3):

$$M' = \begin{pmatrix} 1/t' & r'^*/t'^* \\ r'/t' & 1/t'^* \end{pmatrix} .$$
(9)

it is possible to calculate the transmission t' and reflection r' coefficients for limited periodical medium:

$$\frac{1}{t'} = \frac{1}{T_{01}} \left\{ \frac{1}{t_N} + 2iIm\left(\frac{r_N}{t_N}\right) r_{01} - \frac{R_{01}}{t_N^*} \right\},$$

$$\frac{r'}{t'} = \frac{1}{T_{01}} \left\{ \frac{r_N}{t_N} - \frac{r_N^*}{t_N^*} R_{01} + 2ir_{01}Im\left(\frac{1}{t_N}\right) \right\}.$$
(10)

$$\frac{1}{t_N} = \frac{1}{t \sin\beta} \left(\sin N\beta - t \sin \left(N - 1 \right) \beta \right) ,$$
$$\frac{r_N}{t_N} = \frac{r}{t} \frac{\sin N\beta}{\sin\beta} , \qquad (11)$$

where value β is determined by eq. (8). For φ Bessel beams

$$T_{01} = \frac{4n_0n_1\cos\gamma_0\cos\gamma_1}{n_0\cos\gamma_0 + n_1\cos\gamma_1} ,$$

$$r_{01} = \frac{n_0\cos\gamma_0 - n_1\cos\gamma_1}{n_0\cos\gamma_0 + n_1\cos\gamma_1} ,$$

$$R_{01} = |r_{01}|^2 .$$
(12)

For ρ Bessel light beams in eq. (12) it should be replaced $n_i \to 1/n_i$.

How it follows from obtained correlations, spectral transmission (reflection) dependences are different for φ and ρ BLBs. Then if incident light beam has frequency near the border of photonic band gap, transmission of ρ BLB and reflection of φ polarized beam take place. Given peculiarity can be assumed on the basis of the method for formation of azimuthally and radially polarized Bessel light beams and their spatial separation. However, controlling polarization of forming light fields is of interest.

Let us assume now that a layer with refraction index n_2 has nonlinear properties. Suppose that on the structure both Bessel light beam and power controlling pulse with carrier frequency into the band gap fall. Refraction index of the nonlinear layer is modified under the influence of optical field:

$$n_2^{eff} = \left[n_2^2 + 4\pi \chi_2^{(3)} \left| E \right|^2 \right]^{1/2} \approx n_2 + n_2^{(2)} I \quad .$$
(13)

Here $\chi_2^{(3)}$ is cubic susceptibility of corresponding layer, $n_2^2 = 16\pi^2 \chi_2^{(3)}/(cn_2^2)$ is parameter of nonlinearity for the medium, $I = (c/8\pi)n_2|E|^2$ is intensity of radiation. Influence of light field of the pulse causes the change of refraction index of nonlinear layer in the unit cell and, hence, the form of spectral dependence of transmission (reflection) for φ and ρ polarized beams. By this, for determined wavelength one can achieve coincidence of transmission maxima for φ BLB without pulse and ρ BLB at the presence of controlling pulse. This property can be assumed as a basis of elaboration of switch of BLB polarization.

3. Numerical calculation of passing BLB through out layered - periodical medium

Using above-obtained expressions, we will investigate the peculiarities of polarization transformation of BLB in layered- periodical medium having alternate layers of dielectrics TiO₂ $(n=2.7, d_1 = 59 nm)$ and 9-BCMU $(n=1.6, n^{(2)}=2x10^{-10} cm^2/W, d_2 = 100 nm)$ [16]. The characteristic spectral dependences of transmission coefficient for azimuthally and radially polarized Bessel light beams are shown in fig.1. As it can be seen, the transmitted beam is (quasi) azimuthally polarized at the wavelength $\lambda = 0.76 \mu m$.

Let controlling pulse having wavelength $\lambda_p = 0.64 \,\mu m$, corresponding to the middle of band gap, falls on the structure. We will suppose that pulse energy is $W = 1 \,\mu J$, its duration is $\tau = 0,2 \,ps$ and square of its cross section is $S = 1 \,mm^2$. Then intensity of the pulse is $I = 5x10^8 \,W/cm^2$. It should be noted, that optical distraction of 9-BCMU takes place at the intensity of 1 GW/cm^2 [17].

As it can be seen from fig.2, at the presence of controlling pulse, the Bessel beam transmitted layered- periodical medium is radially polarized. It should be noted, that by varying conicity angle in the range of about 2° , the curve of dependence in Fig.2a,b does not undergo essential changes (fig.3). Thus, proposed method of formation and transformation of azimuthally and radially polarized Bessel light beams is not critical to change of aperture of incident BLBs.

The scheme of spatial separation of circularly polarized incident BLB on azimuthally and radially polarized Bessel beams and their mutual transformation is presented in fig.4.

Here incident circularly polarized Gaussian beam transforms by axicon A_1 into BLB of zeroth order. A beam, transmitting through the periodical medium PS, has determined type of polarization (for example, φ) and is transformed by axicon A_2 into Gaussian beam. Reflected Bessel beam having ρ polarization, is transformed by axicon A_1 back into the beam of Gaussian



FIG. 1. Spectral dependences of the transmission coefficient for the structure $[TiO_2/9 - BCMU]^{16}$ for φ (solid line) and ρ (hatching one) BLB.



FIG. 2. Spectral dependences of transmission coefficient of the structure $[TiO_2/9 - BCMU]^{16}$: a) for ρ (1 - without controlling pulse, 2- at the presence of it) and φ (3 - without controlling pulse, 4 - at the presence of it) BLB; b) for ρ BLB at the presence of the pulse (line 2) and φ BLB without the pulse (line 3). The conicity angle of incident Bessel beam is 27°.

type and goes out optical system with the help of dividing cube. Two beams have equal power owing to circular polarization of incident field. Turning on controlling pulse causes the change of polarization of spatial separation BLBs, namely: at exit of periodical structure we have ρ polarized beam, and the beam, derived from the system with the help of dividing cube, is φ polarized.

4. Conclusion

Thus in the report we have suggested a new method for the formation and mutual transformation of azimuthally and radially polarized BLBs. The analytical and numerical calculation of propagation of BLBs in layered- periodical structure has been realized. It has been shown that on the basis of these structures some effective and compact polarizers and switches of



FIG. 3. Spectral dependences of transmission coefficient for φ BLB without the controlling pulse (line 1) and ρ BLB in the case of incidence of the controlling pulse on the structure $[TiO_2/9 - BCMU]^{16}$ (line 2). The conicity angle of incident Bessel beam is 25° .



FIG. 4. Optical scheme of formation and transformation of azimuthally and radially polarized BLBs. $A_{1,2}$ are axicons; PS is periodical medium, Laser is generator of controlling pulse, M is mirror totally reflecting on the wavelength of laser pulse and totally transmitting on the wavelength of BLB.

polarization for Bessel light beams can be elaborated.

It should be noted that suggested method permits one not only to separate the beams with φ and ρ polarization, but also to form their any superposition. This is achieved by the change of the power of the controlling pulse. It results in rebuilding the transmission spectra for φ and ρ waves and, as a consequence, control over the ellipticity of reflected and transmitted Bessel beams.

References

- [1] R. Dorn, S. Quabis, G. Leuchs. Phys.Rev.Lett. 91 233901-233905 (2003)
- [2] J.. Arlt, Garces- Chavez, Sibbett e.a. Opt. Comm. **197** 239-144 (2001)
- [3] L. Novotny, M.R. Beverslius, K.S. Youngworth, T.G. Brown Phys. Rev. Lett. 86 5251-5255 (2001)

- [4] K.T. Gahagan, G.A. Swartzlander. JOSA. **B16** 533-539 (1999)
- [5] T. Kuga, Y. Torii, N. Shiokawa e.a. Phys.Rev.Lett. 78 4713-4717 (1997)
- [6] S. Brinkman, R. Schreiner, T. Dresel, J. Schwider. Opt. Eng. 37 2506-2511 (1998)
- [7] J.F. de Boer, T.E. Milner, J.S. Nelson. Opt.Lett. 24 300-304 (1999)
- [8] F.P. Schafer. Appl.Phys. **B39** 1-5 (1986)
- [9] S.C. Tidwell, D.H. Ford, W.D. Kimura. Appl. Opt. **29** 2234-2239 (1990)
- [10] J.J. Wynne. IEEE Journal of Quantum Electronics. QE-10 125-129 (1974)
- [11] M.E. Maric, E. Garmire. Appl.Phys.Lett. **38** 743-748 (1981)
- [12] D.J. Armstrong, M.C. Phillips, A.V. Smith. Appl. Optics. 42 3550-3555 (2003)
- [13] A.V. Nesterov e.a. J.Phys.D: Appl. Phys. **32** 2871-2876 (1999)
- [14] S.N. Kurilkina, N.S. Kazak, V.N. Belyi, N.A.Khilo. Physics, chemistry and application of nanostructures. (Singapoor, 2005, P.212-215).
- [15] J.M. Bendickson, J.P. Dowling. Phys. Review E. 53 4107-4112 (1996)
- [16] S. Molyneux, A.K. Kar, B.S. Wherrett e.a. Opt.Lett. 18 2093-2098 (1993)
- [17] P.D. Townsend, J.L. Jackel, G.L. Baker e.a. Appl. Phys. Lett. 55 1829-1834 (1989)